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COOMET	Guidelines for data evaluation of COOMET key comparison	

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INTRODUCTION

The aim of the regional key comparison (RMO KC) is to extend the metrological equivalence over the measurement standards of national metrology institutes (NMIs), which did not take part in CIPM KC [1].

The degree of equivalence of measurement standards of the NMIs participating in key comparisons of a Regional Metrology Organization (RMO), is determined in accordance with section T4 of the Technical Supplement to the Mutual Recognition Arrangement (MRA) with respect to a reference value of CIPM KCRV, using the results of measurements obtained at the NMIs that participated in both comparisons (linking NMIs).

The procedures used for RMO KC data evaluation is intended to provide linking to CIPM KC data with low uncertainty and they should correspond to those used for CIPM KC data evaluation.

The BIPM Director's Advisory Group on Uncertainties worked out the procedure for CIPM KC data evaluation [2] which permits to calculate a KCRV and degree of equivalence of national measurement standards, with respect to the KCRV, as well as the degree of pair equivalence of the national measurement standards, provided that the following conditions are met:

- the travelling measurement standard is stable,
- the measurement results presented by NMIs are reciprocally independent, and
- the Gaussian distribution is assigned to a measurand in each NMI.

Furthermore, in [2] a procedure of evaluating CIPM KC data when the last of the above conditions is not satisfied, is considered.

This guidelines are consistent with [2] with respect to conditions of use and KCRV estimate provided by the weighted mean of CIPM KC data. The algorithms used for data evaluation in this guidelines provide:

- Transformed data of RMO KC with associated uncertainties,
- Degree of equivalence with associated uncertainties for RMO KC participants except for linking NMIs.

2. CONDITIONS OF USE

The procedures suggested have to be applied to evaluation of RMO KC data when the following conditions are satisfied:

a) The CIPM KCRV is determined as a weighted mean in accordance with Procedure A in [2].
b) The travelling measurement standard is stable.
c) Each of the NMIs participating in the RMO KC, has carried out measurements of the travelling standard and submitted their results and corresponding standard uncertainties (and the uncertainty budget)
d) A Gaussian distribution can be assigned to the measurand.

3. RATIONALE

These Guidelines includes two procedures of data evaluation designated by “C” and “D” which conventionally correspond to two types of comparison and ways for RMO KC data transformation. In order to the comparison of RMO and CIPM KC data would be meaningful, RMO data should be transformed. The transformation taking into account the difference in measurands of the both comparisons can be realized by additive correction or by a factor.

The “C” procedure can be applied to those comparisons that require determination of a physical quantity value and assignment of this value to the material measure. In the appropriate CIPM and RMO key comparisons, the material measures with close but, nevertheless, different values of a physical quantity can be used as travelling standards. The “C” procedure requires application of an additive correction for RMO KC data. It is assumed that measurement uncertainties associated with the results of linking NMIs, obtained in the CIPM KC and RMO KC, remain the same.

The “D” procedure can be applied to those comparisons, in which measurands differ greatly. I can occur in the case when the participants determine a value of the calibration coefficient for one and the same measuring instrument used as the travelling standard. The “D” procedure requires the RMO KC data transformation by a factor. It is assumed that the relative measurement uncertainties associated with the results of linking NMIs, obtained in the CIPM KC and RMO KC, remain the same.

The Guidelines deals with two practical cases

- The data of CIPM KC and RMO KC are not correlated except for data of the linking NMI. The strong correlation between the results of linking NMIs is assumed. If the correlation coefficients are less then 0.5 the procedure described in [3] is more preferable.
- Some results of the RMO KC and CIPM KC are correlated due to their traceability

The inference of all equations is given in *Appendix A.*

The *Appendix B* includes the procedure for RMO KC data evaluation when degrees of equivalence are expressed in relative form.

4. NOTATIONS

- x_{ref} - CIPM KCRV;

$$x_{ref} = \frac{\sum_1^N \frac{x_i}{u^2(x_i)}}{\sum_1^N \frac{1}{u^2(x_i)}}, \qquad u^2(x_{ref}) = \frac{1}{\sum_1^N \frac{1}{u^2(x_i)}}$$

- x_i - the results of the CIPM KC;
- $\tilde{x}_i$ - the results of the RMO KC;
- x_i^*, $\tilde{x}_i^*$ - the results of the linking NMI obtained in the CIPM and RMO KC respectably;
- $\tilde{x}_i'$ - transformed data of the RMO KC
- N – a number of participants of the CIPM KC;
- N_1 - a number of participants of the RMO KC;
- L - a number of the linking NMIs. For convenience the results of the linking NMIs are numbered from 1 to L in both comparisons;
- $u(\bullet)$ - standard uncertainty
- S_k - standard deviation of the results of $k-$th linking NMI that obtained in conditions of intermediate precision

- ρ_k - correlation coefficient of the results of the $k-$th linking NMI
- r_{ij} -coefficient of correlation of the results of $i-$th NMI and j -th NMI
- $\mathrm{cov}(x_i, x_j)$ - covariance of the results of $i-$th NMI and j -th NMI
- CMC - калибровочные и измерительные возможности

Subscript “*rel”* means that the corresponding characteristic is given a relative form

5. PROCEDURE C.

5.1 Transformed data

Transformed data are given by

$$\tilde{x}_i' = \tilde{x}_i + \Delta \qquad (1)$$

with associated standard uncertainty

$$u^2(\tilde{x}_i') = u^2(\tilde{x}_i) + u^2(\Delta) \qquad (2)$$

where

$\tilde{x}_i', \quad i > L$ - transformed data with the exception of data of linking NMI;

Δ - additive correction estimated in 5.1.1. and 5.1.2.

5.1.1. Correction estimate in case of one linking NMI

In case of one linking NMI the correction with associated uncertainty is given by

$$\Delta = x^* - \tilde{x}^*, \quad u^2(\Delta) = 2S^2, \qquad (3)$$

where $x^*, \tilde{x}^*$ the results of the linking NMI obtained in CIPM and RMO KC respectably, S - standard deviation of intermediate precision.

Note S can be calculated analytically using ρ by $S^2 = (1-\rho)u^2(x^*)$, under assumption $u(x^*) = u(\tilde{x}^*)$.

5.1.2. Correction estimate in case of a number of linking NMIs

In case of a number of linking NMIs the correction is a weighted mean of correction estimates based on results of every linking NMI

$$\Delta = \frac{\sum_{k=1}^{L} \frac{\Delta_k}{S_k^2}}{\sum_{k=1}^{L} S_k^{-2}}, \quad u^2(\Delta) = \frac{2}{\sum_{k=1}^{L} S_k^{-2}} \qquad (4),$$

where $\Delta_k = x_k^* - \tilde{x}_k^*$,

5.2 Degrees of equivalence

Degree of equivalence of the i-th NMI is estimated by

$$d_i = \tilde{x}_i + \Delta - x_{ref} \qquad (5)$$

with associated standard uncertainty

$$u^2(d_i) = u^2(\tilde{x}_i) + u^2(x_{ref}) + u^2(\Delta)\left\{1 - u^2\left(x_{ref}\right) \times \sum_1^L u^{-2}\left(x_i^*\right)\right\}, \qquad (6)$$

in case of independent data , and

$$u^2(d_i) \cong u^2(\tilde{x}_i) - u^2(x_{ref}) + u^2(\Delta)\left\{1 - u^2\left(x_{ref}\right) \times \sum_1^L u^{-2}\left(x_i^*\right)\right\} \qquad (7)$$

if *i-th* result is traceable to results of any participants of CIPM KC.

NOTE *Formula (5) can be rewritten as follows*

$$d_i = \sum_{k=1}^{L} \omega_k d_{ik} + \sum_{k=1}^{L} \omega_k d_k^*$$

where $\omega_k = \dfrac{\frac{1}{S_k^2}}{\sum_{k=1}^{L} S_k^{-2}}$,

$d_{ik} = \tilde{x}_i - \tilde{x}_k^*$ - pairwise degree of equivalence of RMO KC participant and k-th linking institute (cl. 5.3.1)

d_k^* - degree of equivalence of CIPM KC participant

5.3 Pair degrees of equivalence

5.3.1 Pair degrees of equivalence for participants of the RMO KC

The pair degree of equivalence is estimated by

$$d_{ij} = d_i - d_j = \tilde{x}_i - \tilde{x}_j,$$
$$u^2(d_{ij}) = u^2(\tilde{x}_i) + u^2(\tilde{x}_j) - 2\,\mathrm{cov}(\tilde{x}_i, \tilde{x}_j). \qquad (8)$$

5.3.2 Pair degrees of equivalence for participants of the RMO KC and CIPM KC

The pair degree of equivalence is estimated by

$$d_{ij} = \tilde{x}_i + \Delta - x_j \qquad (9)$$

with associated standard uncertainty

$$u^2(d_{ij}) = u^2(\tilde{x}_i) + u^2(\Delta) + u^2(x_j) - 2\,\mathrm{cov}(\tilde{x}_i, x_j), \qquad (10)$$

for the case when the participant of CIPM KC is not a linking NMI.

For the case when the participant of CIPM KC is the linking NMI the pair degree is estimated by (8)

5.3.3.Correlation

Covariance's are estimated by careful analysis of the uncertainty budget by the pilot NMI.

$$\mathrm{cov}(\tilde{x}_i, \tilde{x}_j) = u_0^2 \text{ (or } \mathrm{cov}(\tilde{x}_i, x_j) = u_0^2\text{)}, \qquad (11)$$

where u_0^2 - common input to the uncertainty budgets of both results.

5.4. Confirmation of the declared uncertainties

The declared uncertainties are judged as confirmed if the following equation is satisfied

$$|d_i| < 2u(d_i) \qquad (12).$$

It actually means confirmation of the corresponding CMC.

6. PROCEDURE D.

6.1 Transformed data

The transformed data and associated relative uncertainties are estimated by

$$\tilde{x}_i' = \tilde{x}_i \times c$$
$$u_{rel}^2(\tilde{x}_i') = u_{rel}^2(\tilde{x}_i) + u_{rel}^2(c) \qquad (13)$$

6.1.1. Transformation factor in case of one linking NMI

In case of one linking NMI the transformation factor and associated relative uncertainty is given by

$$c = \frac{x^*}{\tilde{x}^*}\ , \quad u_{rel}^2\left(\frac{x^*}{\tilde{x}^*}\right) = 2u_{rel}^2\left(x^*\right)\left(1-\rho_i\right) \qquad (14)$$

where $x^*, \tilde{x}^*$ - the results of the linking NMI obtained in both CIPM and RMO KC respectably,

ρ - correlation factor of the results of $i-$th linking NMI

correlation factor for measurement results obtained in linking NMI $u_{rel}\left(x^*\right) = u_{rel}\left(\tilde{x}^*\right)$.

6.1.2 Transformation factor in case of a number of the linking NMIs

In case of a number linking NMIs the transformation factor is given by weighted mean of the estimates based on the results of every linking NMI

$$c = \frac{\sum_{k=1}^{L} \frac{1}{u_{rel}^2\left(x_k^*\right)\left(1-\rho_k\right)} \times \frac{x_k{}^*}{\tilde{x}_k{}^*}}{\sum_{k=1}^{L} \frac{1}{u_{rel}^2\left(x_k^*\right)\left(1-\rho_k\right)}} \qquad u_{rel}^2(c) = \frac{2}{\sum_{k=1}^{L} \frac{1}{u_{rel}^2\left(x_k^*\right)\left(1-\rho_k\right)}} \qquad (15)$$

6.2 Degrees of equivalence

The degree of equivalence for *i*-th NMI is estimated by

$$d_i = \tilde{x}_i \times c - x_{ref} \qquad (16)$$

with the associated standard uncertainty

$$u^2\left(d_i\right) = c^2 u^2\left(\tilde{x}_i\right) + u^2\left(x_{ref}\right) + 2\left(\sum_{1}^{L} \frac{1}{u^2\left(x_k^*\right)\left(1-\rho_k\right)}\right)^{-1} \left(1 - u^2\left(x_{ref}\right) \sum_{1}^{L} \frac{1}{u^2\left(x_k^*\right)}\right), \qquad (17)$$

if the result from *i-th* NMI are not correlated with the results of CIPM KC, and

$$u^2\left(d_i\right) \cong c^2 u^2\left(\tilde{x}_i\right) - u^2\left(x_{ref}\right) + 2\left(\sum_{1}^{L} \frac{1}{u^2\left(x_k^*\right)\left(1-\rho_k\right)}\right)^{-1} \left(1 - u^2\left(x_{ref}\right) \sum_{1}^{L} \frac{1}{u^2\left(x_k^*\right)}\right) \qquad (18)$$

if the result from *i-th* NMI are traceable to any result of CIPM KC

Note In some practical cases it can be convenient to express the degree of equivalence in relative form $d_{i,rel} = \frac{x_i}{x_{ref}}$. The corresponding procedure for data evaluation is given in Appendix B.

If $d_{i,rel} = \frac{x_i}{x_{ref}}$, formula (16) can be rewritten as follows

$$d_i = \sum_{1}^{L} \omega_k d_{k,rel}^* d_{ik,rel}$$

where

$$\omega_k = \frac{\sum_{k=1}^{L} \frac{1}{u_{rel}^2\left(x_k^*\right)\left(1-\rho_k\right)}}{\sum_{k=1}^{L} \frac{1}{u_{rel}^2\left(x_k^*\right)\left(1-\rho_k\right)}}$$

$d_{k,rel}^*$ - degree of equivalence of *k*-th linking institute in relative form

$d_{ik,rel} = \frac{\tilde{x}_i}{\tilde{x}_k}$ - pairwise degree of equivalence of RMO KC participant and *k*-th linking institute in relative form(B.6)

6.3 Pair degrees of equivalence

6.3.1 Pair degrees of equivalence for participants of the RMO KC

The pair degree of equivalence and associated uncertainty is estimated by

$$d_{ij} = d_i - d_j = c(\tilde{x}_i - \tilde{x}_j),$$
$$u^2(d_{ij}) = c^2\left(u^2(\tilde{x}_i) + u^2(\tilde{x}_j) - 2\operatorname{cov}(x_i, x_j)\right). \qquad (19)$$

6.3.2 Pair degrees of equivalence for participants of the RMO KC and CIPM KC

The pair degree of equivalence is estimated by

$$d_{ij} = \tilde{x}_i \times c - x_j \qquad (20)$$

with associated standard uncertainty

$$u^2(d_{ij}) = c^2u^2(\tilde{x}_i) + u^2(x_j) + \frac{2}{\sum_1^L \frac{1}{u^2(x_k)(1-\rho_k)}} - 2c\operatorname{cov}(\tilde{x}_i, x_j), \qquad (21)$$

for the case when the participant of CIPM KC is not a linking NMI, and

$$u^2(d_{ij}) = c^2u^2(\tilde{x}_i) + u^2(x_j) - 2c\operatorname{cov}(\tilde{x}_i, x_j) \qquad (22)$$

for the case when the participant of CIPM KC is the linking NMI

6.3.3 Correlation

Covariance's are estimated by careful analysis of the uncertainty budget by the pilot NMI. $\operatorname{cov}(\tilde{x}_i, \tilde{x}_j) = u_0^2$ (or $\operatorname{cov}(\tilde{x}_i, x_j) = u_0^2$), where u_0^2 - common input to the uncertainty budgets of both results.

6.4 Confirmation of the declared uncertainties

The declared uncertainties are judged as confirmed if the following equation is satisfied

$$|d_i| < 2u(d_i) \qquad (23)$$

References:

1. Bureau International des Poids et Mesures (BIPM) 1999 Mutual recognition of national standards and of calibration and measurement certificates issued by national metrology institutes *BIPM Publication* (Sevres:BIPM)
2. Cox M G 2002 *Metrologia* **39** 589-95
3. C.Elster, A.Link and W.Woeger 2003 *Metrologia* **40** 189-194

Appendix A Interferences of all equations
Appendix B Relative degree of equivalence