Workshop on OIML G-19:
The role of measurement uncertainty in conformity assessment decisions in legal metrology

Introduction

COOMET TC2 seminar

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Verification authority of Lower Saxony (MEN)





Outline

1. Introduction
2. Introduction into OIML G19
3. Some brief introduction how to calculate measurement uncertainties
4. Some information about useful software to calculate measurement uncertainties

Before we start:

You can get this presentation after the seminar! (about 150 pages)

Will be made available on coomet.net

But of course you may take your own notes.

Before we start:

There is a huge interest in this seminar!

We expected about 15 partipants (from COOMET TC 2), but now there are almost 100 participants …

Why?

Is there some magic in there?

Am I doing anything wrong?

Some introductory remarks:

What is the reason for this seminar?

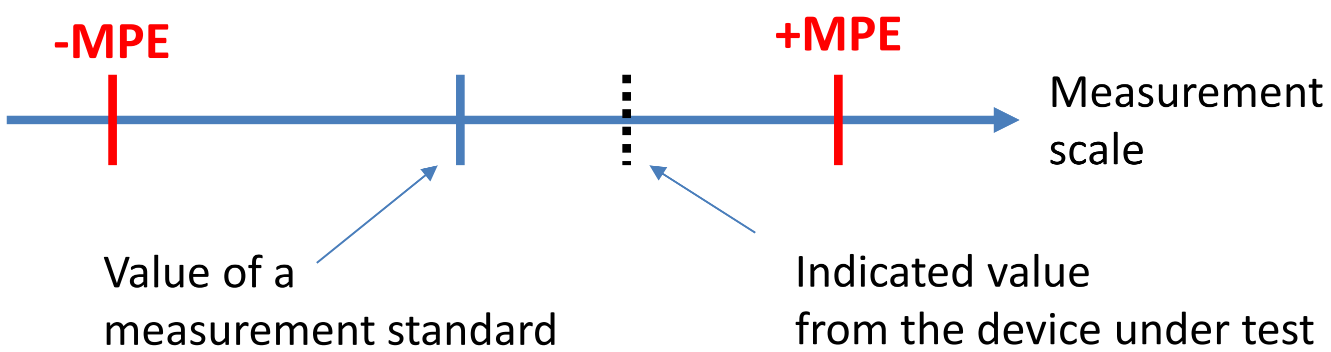
There was interest of the members of COOMET TC 2 „Legal metrology“, because we deal with **conformity assessment in legal metrology, i. e. type approval, verification and market surveillance.**

Is conformity to legislation given?

Some introductory remarks:

What do we do normally?

The **classical approach** is to check, whether a measured value is within the Maximum Permissible Errors (MPE) given by legislation:



But in principle we know this can be wrong!!!

Here OIML G 19 comes in!

We need to deal with measurement uncertainties!

But the knowledge within the auditorium may be totally different!

Most of the colleagues in legal metrology
don‘t deal with measurement uncertainties on a daily basis.
So the intention is to hold this seminar
on a more „basic level“!

What could you „take home“ after this seminar?

- Overview about OIML G 19
- Some basic information about measurement uncertainty
- Some information about calculating the risk to make a wrong decision in conformity assessment
- Some information about useful commercial and free software to calculate measurement uncertainties

1. Some background information on OIML G-19

GUIDE

OIML G 19

Edition 2017 (E)

The role of measurement uncertainty in conformity assessment decisions in legal metrology

Responsibility of OIML TC 3/SC 5

This publication – reference OIML G 19, edition 2017 (E) –

was developed by

TC 3: Metrological Control
SC 5: Conformity Assessment
Project Group 2: Task for a Guide
“Expression of uncertainty in measurement in legal metrology applications.”

TC 3/SC 5: Conformity assessment

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Secretariat

UNITED STATES

Dr. Charles D. Ehrlich

Participating members (27)

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From the chapter „1 Scope and objectives“

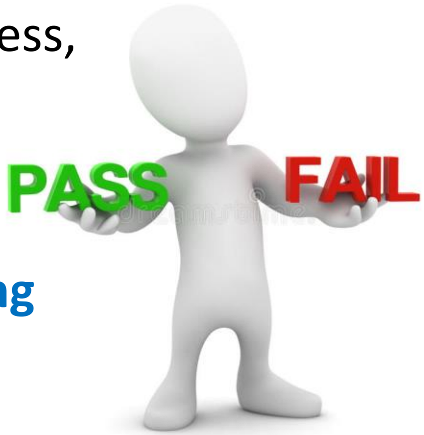
It is assumed that **the reader has at least a general familiarity with the concepts presented in the *Guide to the Expression of Uncertainty in Measurement*** [1] (hereafter denoted by GUM), and possibly also with the concepts in its Supplements [2][3][4][5].

It is anticipated that **this Guide will eventually become an OIML Document**; as such, its contents will need to be incorporated, as appropriate, in OIML Recommendations and Documents. However, this initial version is presented as an OIML Guide in order to give OIML Technical Committees, Subcommittees and Project Groups additional **time to consider the contents and how they can be incorporated into the Recommendations and Documents** for which they are responsible.

From the chapter „1 Scope and objectives“

The main objective of this OIML Guide is **to provide guidance** … how to take measurement uncertainty into account in conformity assessment in legal metrology,

that is, when determining **whether an entity** (product, process, system, person or body) meets relevant standards or **fulfils specified requirements**.



A **particular focus is on conformity assessment of measuring instruments** (or systems), especially when using measured values, obtained **during the testing or verification** of the measuring instruments or systems, **as the basis for making pass-fail decisions** in legal metrology. (Not for prepackages!)

From the chapter „1 Scope and objectives“

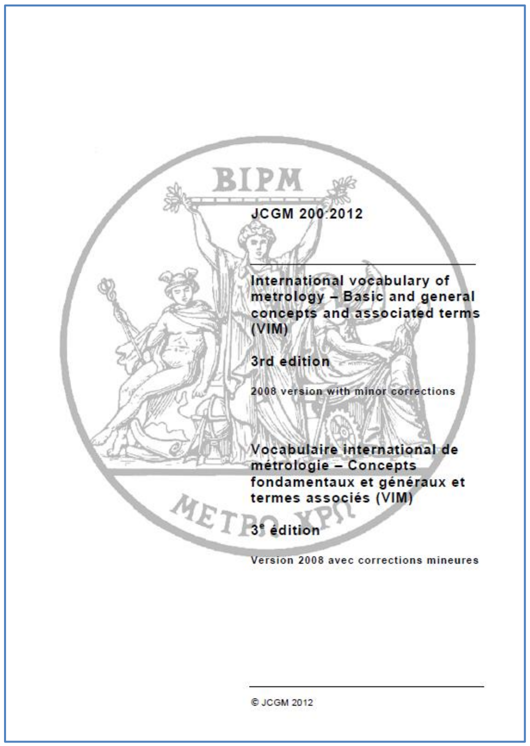
The proposals include providing and referencing information on **how to assess the possible “risks” of erroneous conformity decisions.**

Such **risks arise unavoidably** from the measurement uncertainty associated with the measured values obtained during testing or verification of a measuring instrument or system.

That is, measurement uncertainty in a test result … can be a concern in conformity assessment by inspection since if it is not accounted for **it can lead to incorrect estimates of the consequences** of entity error and increase the risk of making incorrect decisions,
such as failing a conforming entity or passing a nonconforming entity
when the test result is close to a tolerance limit.

From the chapter „2 Terms and definitions“

„VIM“ „VIML“ „GUM“



BIPM

JCGM 200:2012

International vocabulary of metrology – Basic and general concepts and associated terms (VIM)

3rd edition

2008 version with minor corrections

Vocabulaire international de métrologie – Concepts fondamentaux et généraux et termes associés (VIM)

3e édition

Version 2008 avec corrections mineures

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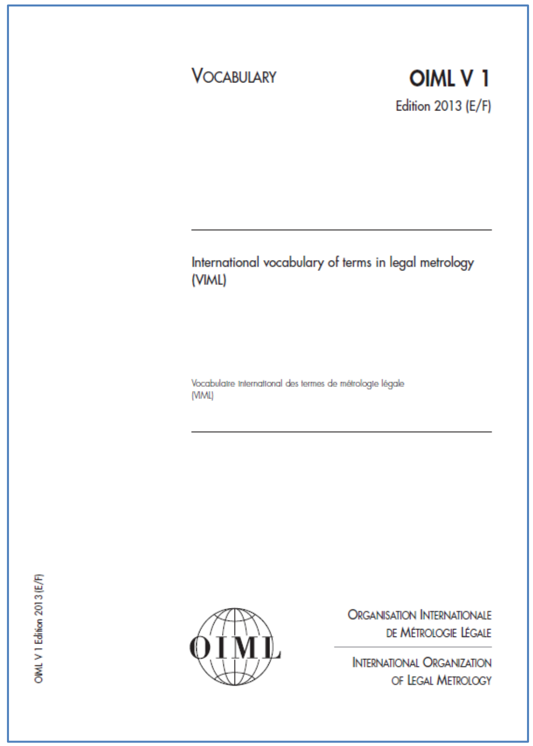


VOCABULARY

OIML V 1

Edition 2013 (E/F)

International vocabulary of terms in legal metrology (VIML)

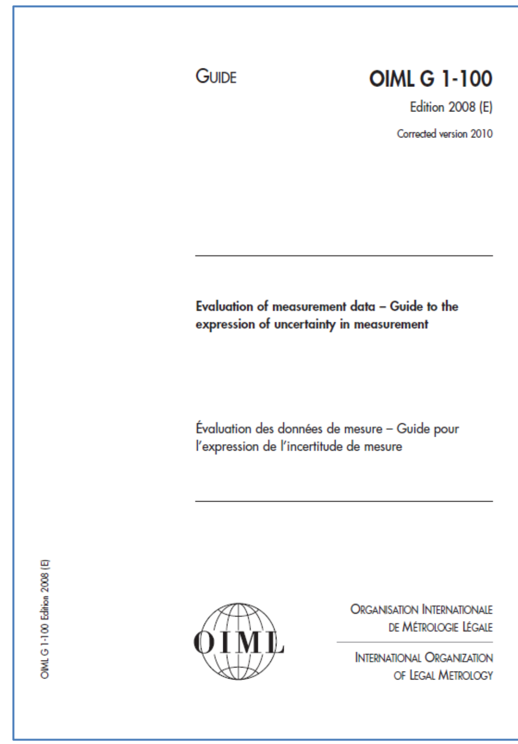
Vocabulaire international des termes de métrologie légale (VIML)

OIML

ORGANISATION INTERNATIONALE DE MÉTROLOGIE LÉGALE

INTERNATIONAL ORGANIZATION OF LEGAL METROLOGY

OIML V 1 Edition 2013 (E/F)



GUIDE

OIML G 1-100

Edition 2008 (E)

Corrected version 2010

Evaluation of measurement data – Guide to the expression of uncertainty in measurement

Évaluation des données de mesure – Guide pour l'expression de l'incertitude de mesure

OIML

ORGANISATION INTERNATIONALE DE MÉTROLOGIE LÉGALE

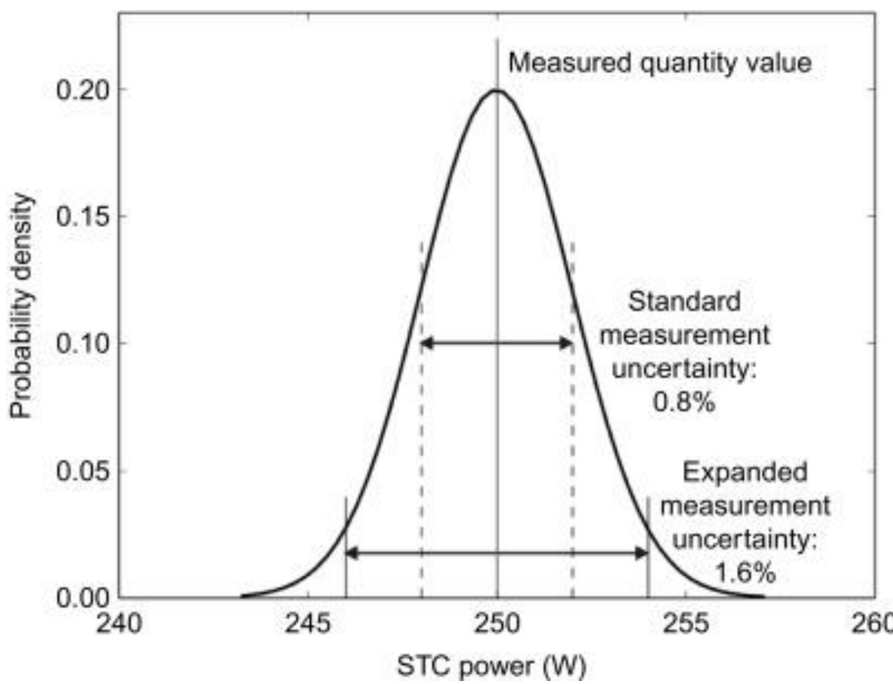
INTERNATIONAL ORGANIZATION OF LEGAL METROLOGY

OIML G 1-100 Edition 2008 (E)

Everything can be **downloaded free of charge** from the OIML webpage!

From the chapter „3 Introduction“

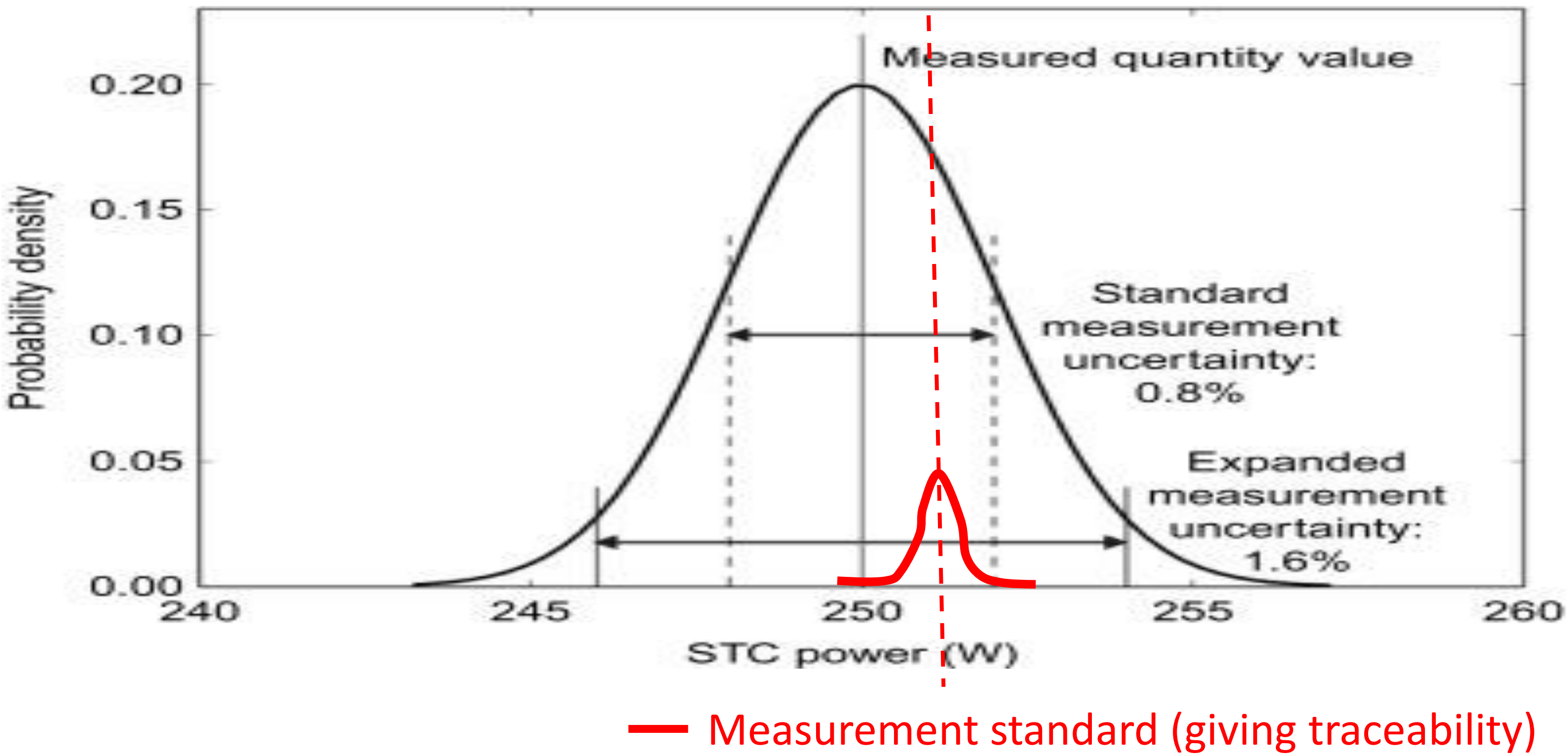
Measured (indicated) value and measurement uncertainty



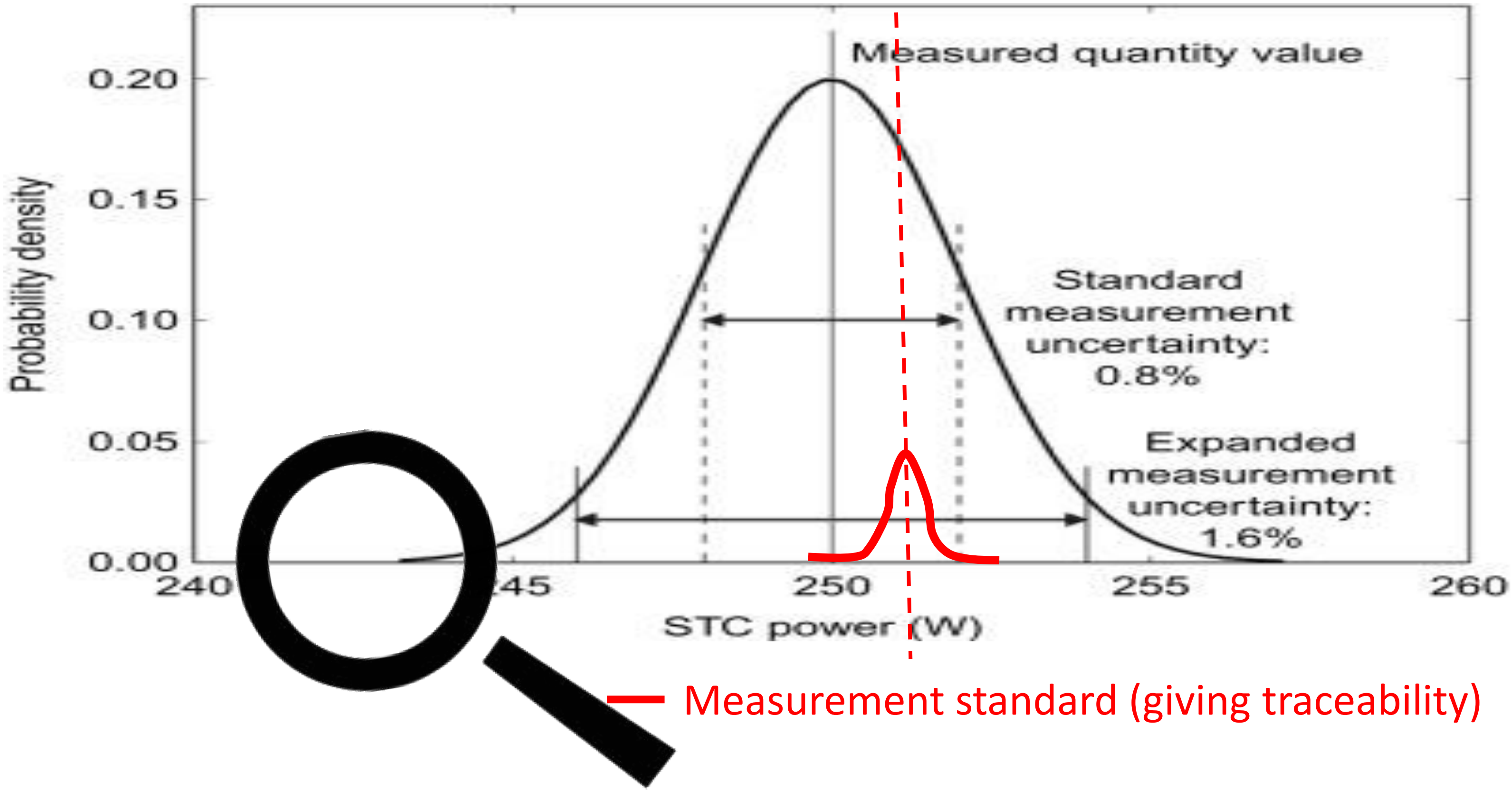
In case of repeated measurements!

From the chapter „3 Introduction“

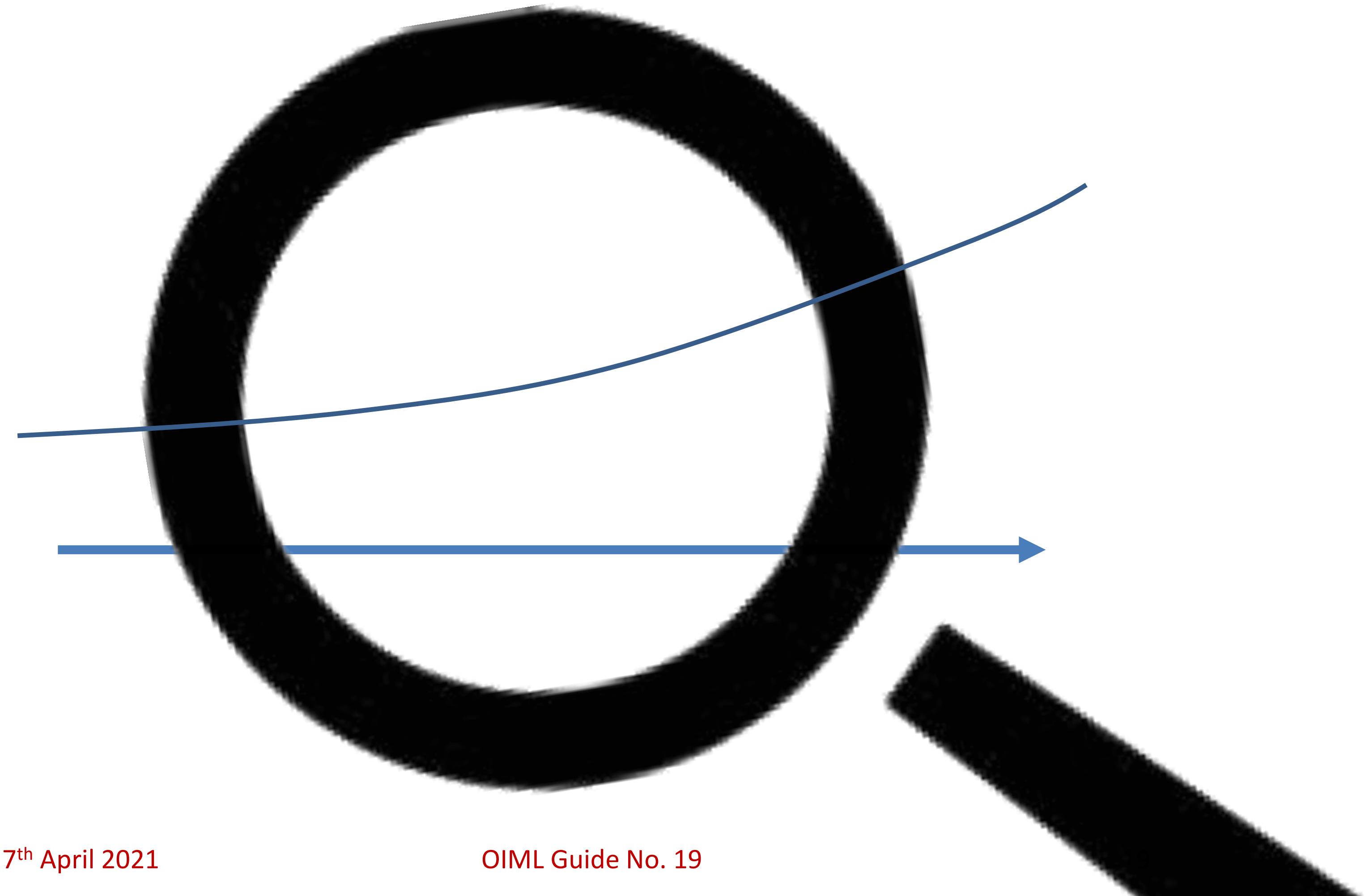
„Calibration“ ➡ The conventional use of the concept of measurement uncertainty.



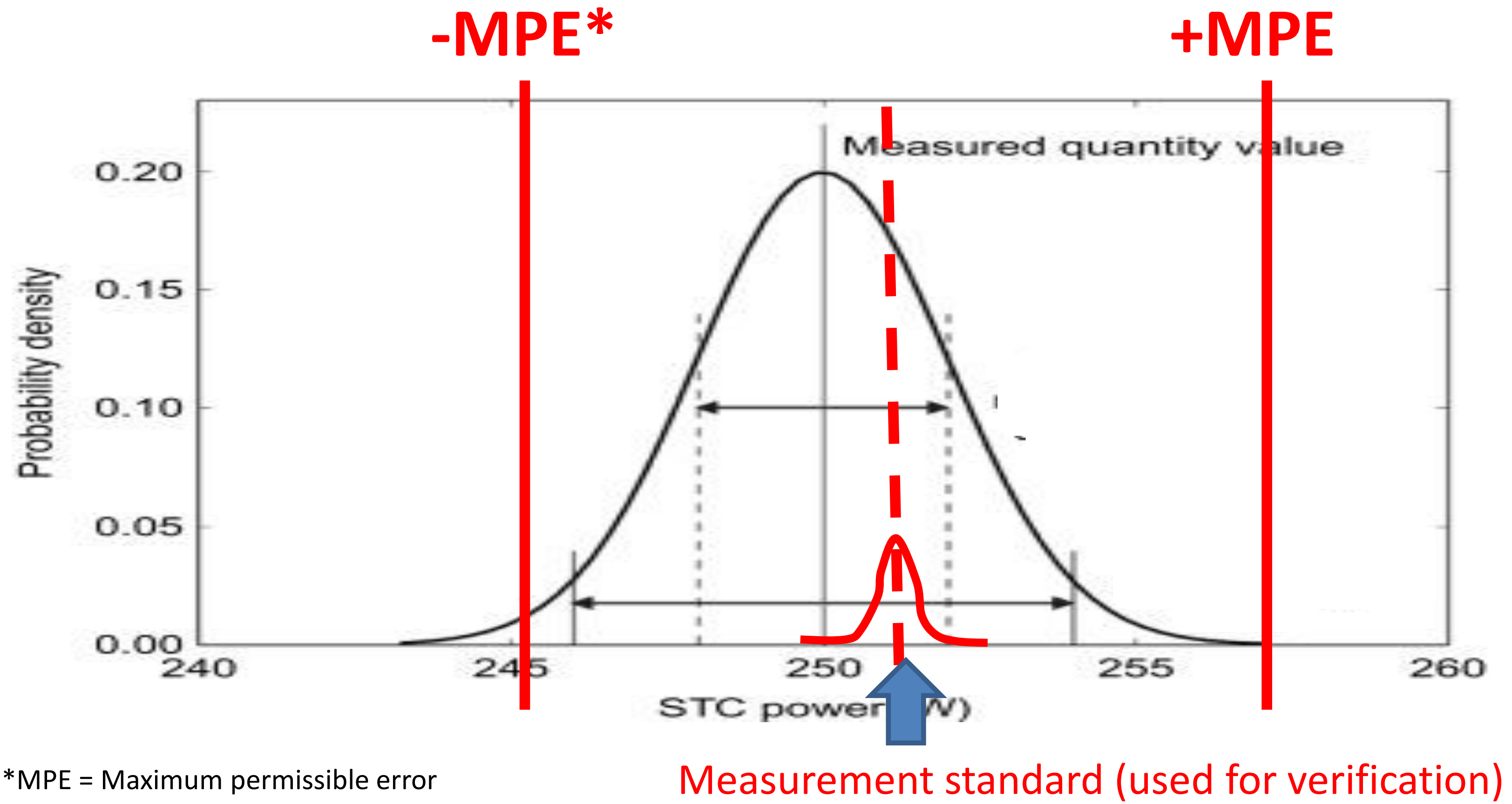
Important: Probability is never zero!



Important: Probability is never zero!



Measurement uncertainty in case of verification:

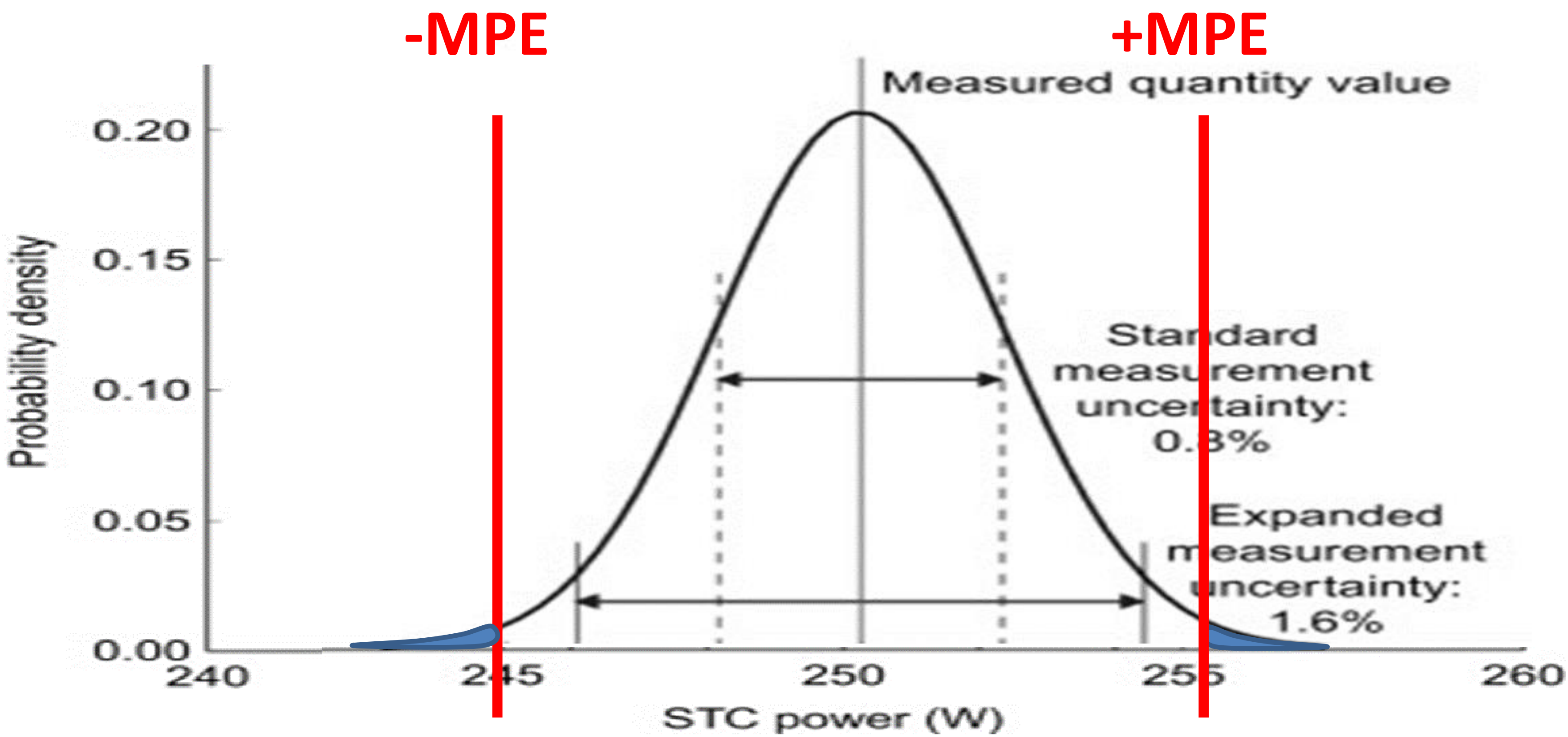


*MPE = Maximum permissible error

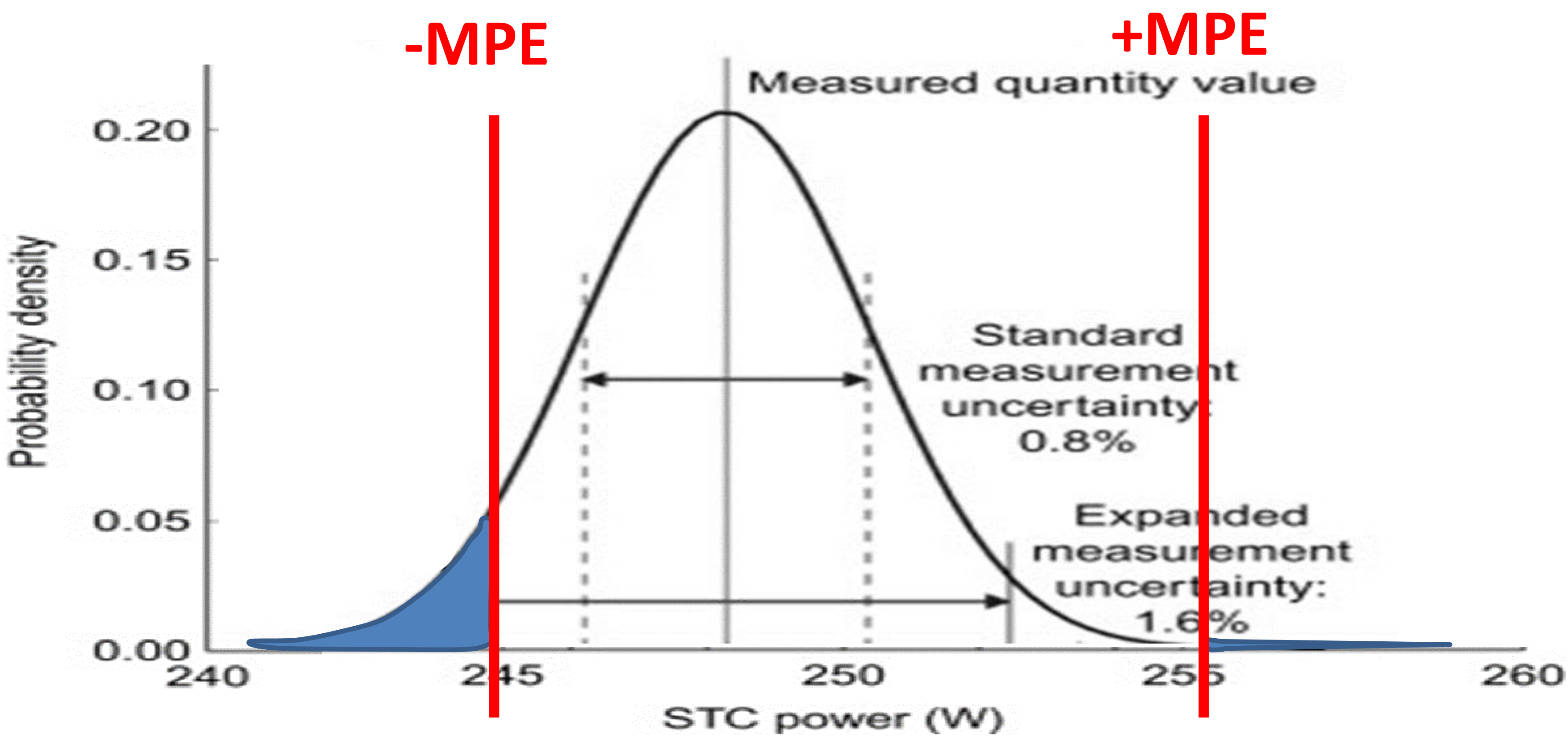
Questions arise:

- Is a measuring instrument under test really conform?
- What situation is acceptable? What is non-acceptable?
- What is the risk for passing the test, when it is non-conform?
- What is the risk for failing the test, when it is conform?

Risks: Example 1 = small risk

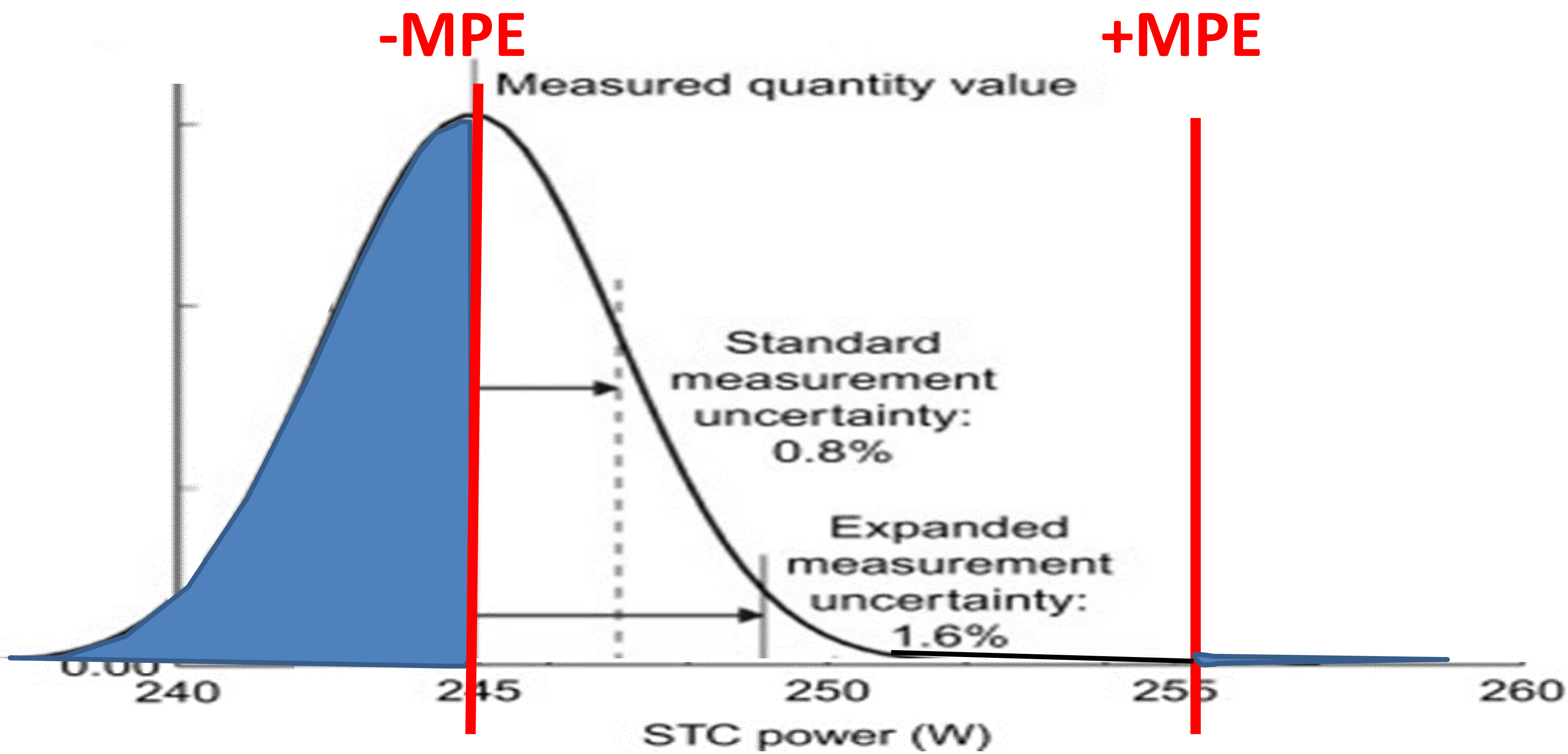


Risks: Example 2 = higher risk



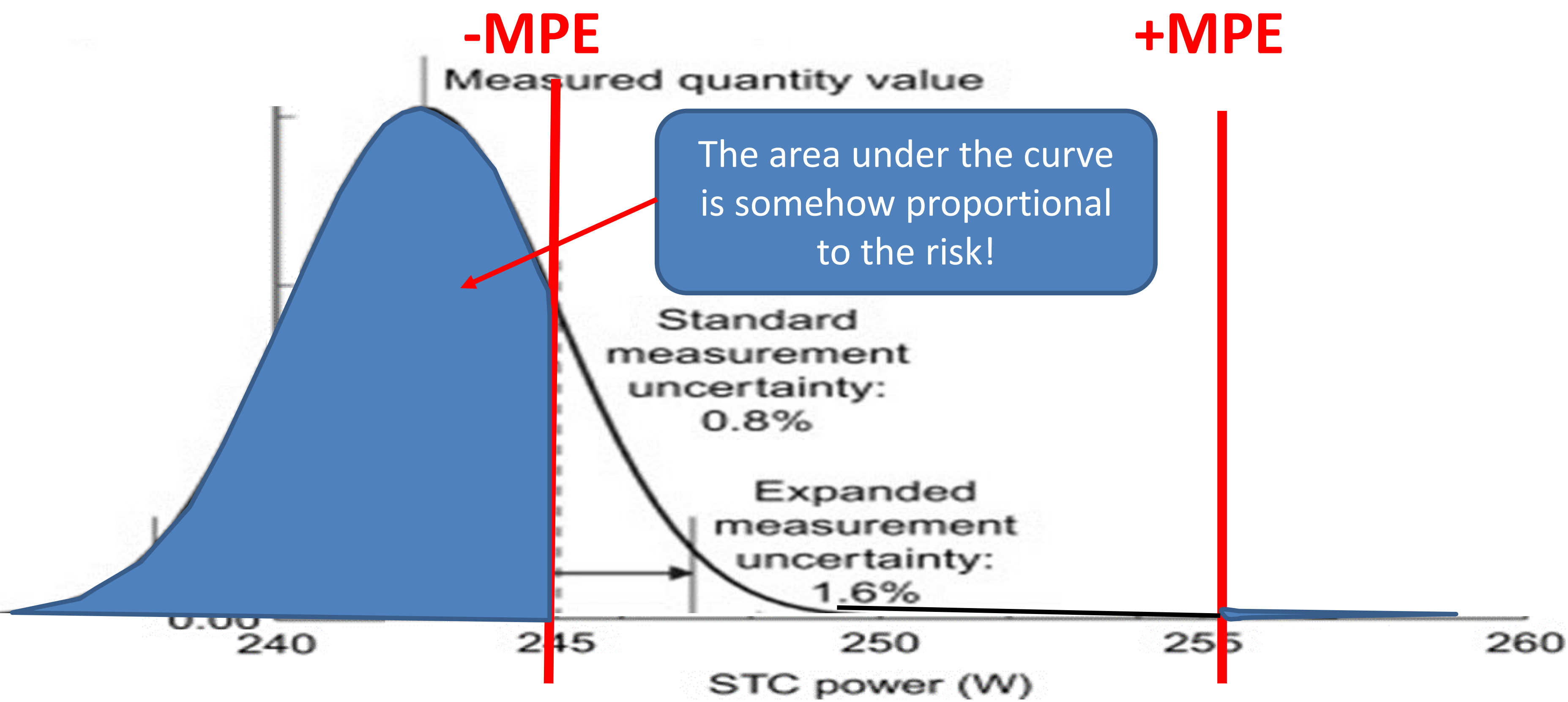
Mean value within the MPEs ➡ Instrument is considered to be conform!

Risks: Example 3 = big risk = approx. 50%



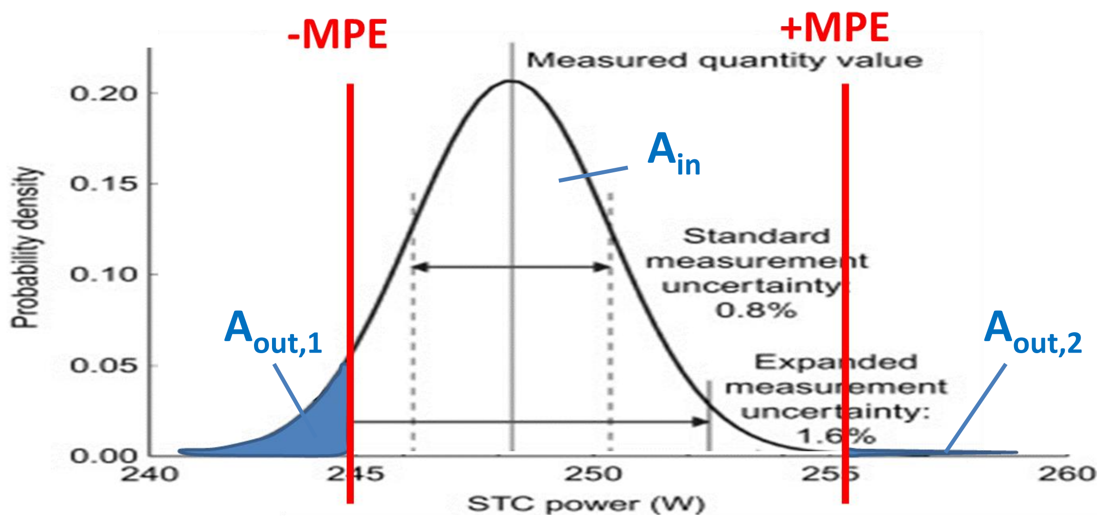
Conform or not conform? That‘s the question!

Risks: Example 4 = very big risk = >50%



Mean value outside the MPEs ⇨ Instrument is considered to be non-conform!

Risk calculation: „Instrument is conform“

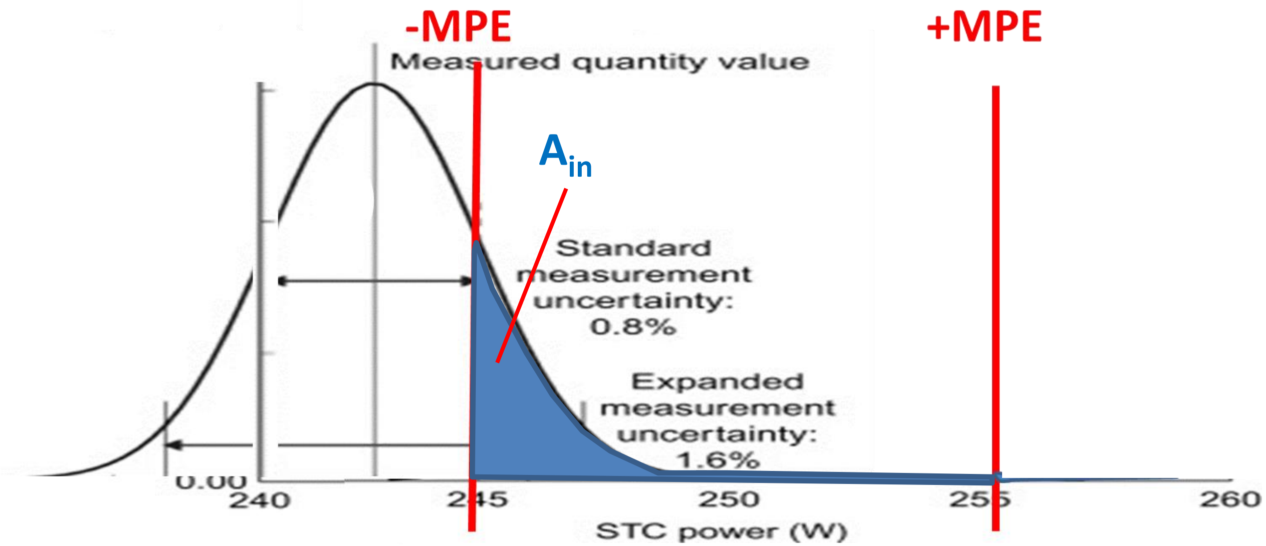


Definition:

Risk of a conform instrument being non-conform resp. of a conform instrument failing the test:

$$\text{Risk} = \frac{A_{out,1} + A_{out,2}}{A_{out,1} + A_{out,2} + A_{in}} = \frac{A_{out}}{A_{total}}$$

Risk calculation: „Instrument is non-conform“



Risk of a non-conform instrument being conform resp. of a non-conform instrument passing the test:

$$\text{Risk} = \frac{A_{in}}{A_{out,1} + A_{out,2} + A_{in}} = \frac{A_{in}}{A_{total}}$$

Risk calculation:

During the revision of the ISO/IEC 17025 standard in 2017, the following **new requirement to include decision rules in test reports** was added:

7.8.6 Reporting statements of conformity

7.8.6.1 When a statement of conformity to a specification or standard is provided, the laboratory shall document the decision rule employed, taking into account the level of risk (such as false accept and false reject and statistical assumptions) associated with the decision rule employed, and apply the decision rule.

NOTE Where the decision rule is prescribed by the customer, regulations or normative documents, a further consideration of the level of risk is not necessary.

7.8.6.2 The laboratory shall report on the statement of conformity, such that the statement clearly identifies:

a) to which results the statement of conformity applies;

b) which specifications, standards or parts thereof are met or not met;

c) the decision rule applied (unless it is inherent in the requested specification or standard).

NOTE For further information, see ISO/IEC Guide 98-4.

And a very practical question:

How big should the measurement uncertainty of your measurement standard (e. g. used for verification) be compared with the maximum permissible error?

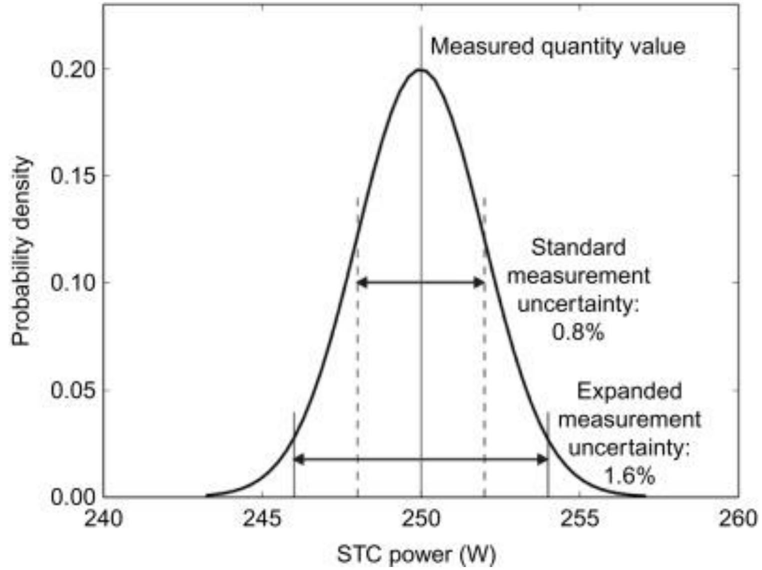
1/3 or 1/4 or 1/5 or 1/10 or….?

⇨ **The uncertainty of the measurement standard has an impact on the risk!**

(⇨ We will come back to this topic a little bit later!)

Annex A: Error and measurement uncertainty

The concept of measurement uncertainty can be described as a measure of **how well the 'true' value of the measurand is believed to be known**.



No „true value“ in this diagram!

It is not possible to know how well the 'true' value of the measurand is known.

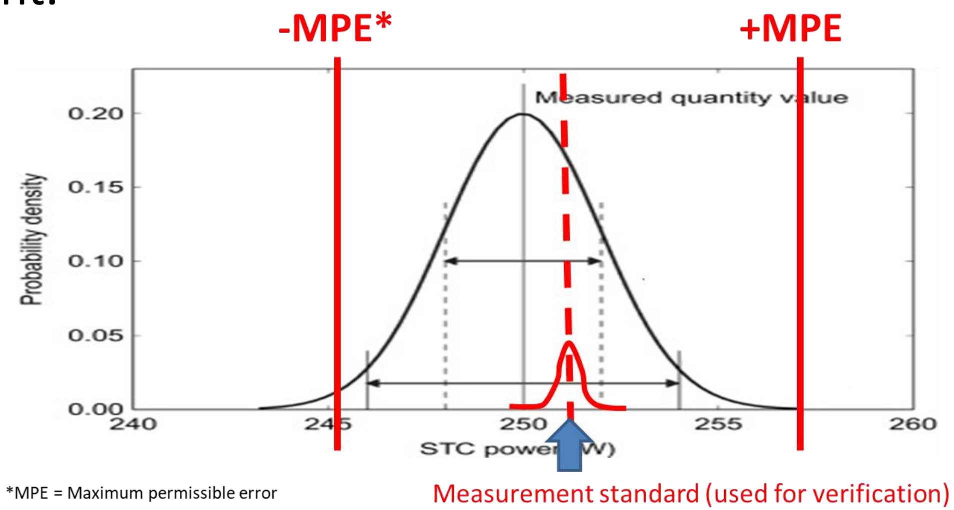
Annex A: Error and measurement uncertainty

When **making decisions in legal metrology** about whether measuring systems are performing according to specified requirements, if the GUM approach is to be followed **it becomes necessary to make such decisions on a probabilistic basis**.

Testing or verifying in this context means
=> that a **decision is being made** about
=> whether the measuring system under test
=> **is providing indicated values of a quantity** being measured
=> that are **believed to be 'close enough' to the 'true' value**,
=> as determined by using measurement standards,
=> for the regulatory purpose at hand.

Annex A: Error and measurement uncertainty

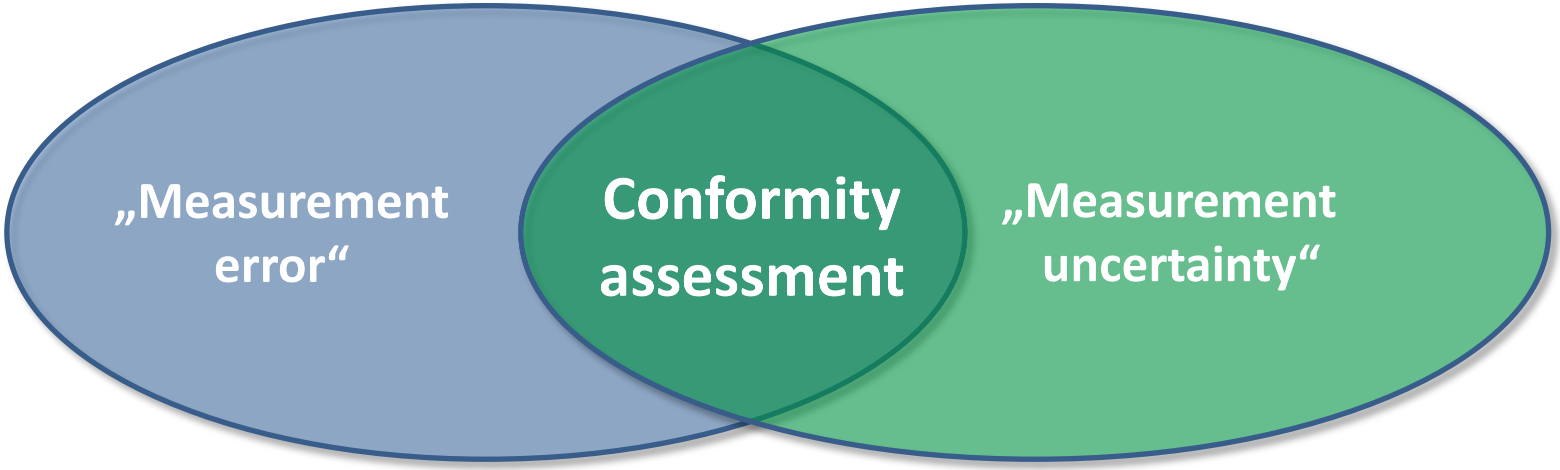
Using the GUM approach, the **objective of verification** then becomes **to determine the degree of belief** (level of confidence) that the **'true' value of the 'error of indication' lies within the maximum permissible errors** when taking measurement uncertainty (of the 'error of indication'!) into account.



*MPE = Maximum permissible error

Frequent question:

How can “measurement error” and “measurement uncertainty” coexist when considering measurement in the context of conformity assessment?



Questions?

Remarks?

How can "measurement error" and "measurement uncertainty" coexist when considering measurement in the context of verification?

Conformity assessment in legal metrology typically involves
comparing
the **measured error of indication** of a measuring instrument or system
to an MPE that is specified in a legal regulation.

The **error of indication** is typically calculated in legal metrology
as the **difference**
between the **indicated value** and **a value as given by a measurement standard**.

Measurement uncertainty!

Measurement Error: Example for Standard Weight

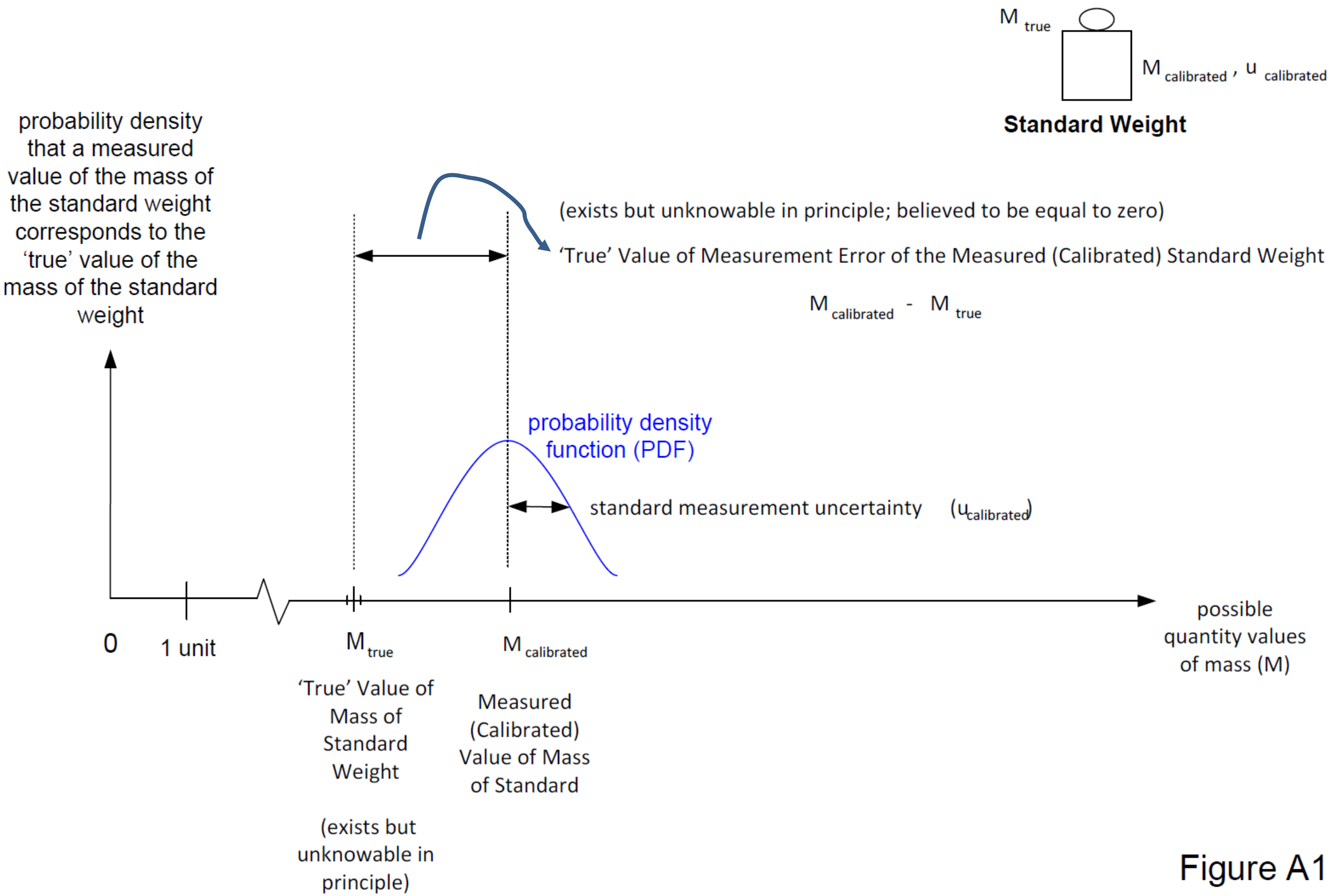


Figure A1

Error of Indication: Example for Weighing Instrument (Under Test)

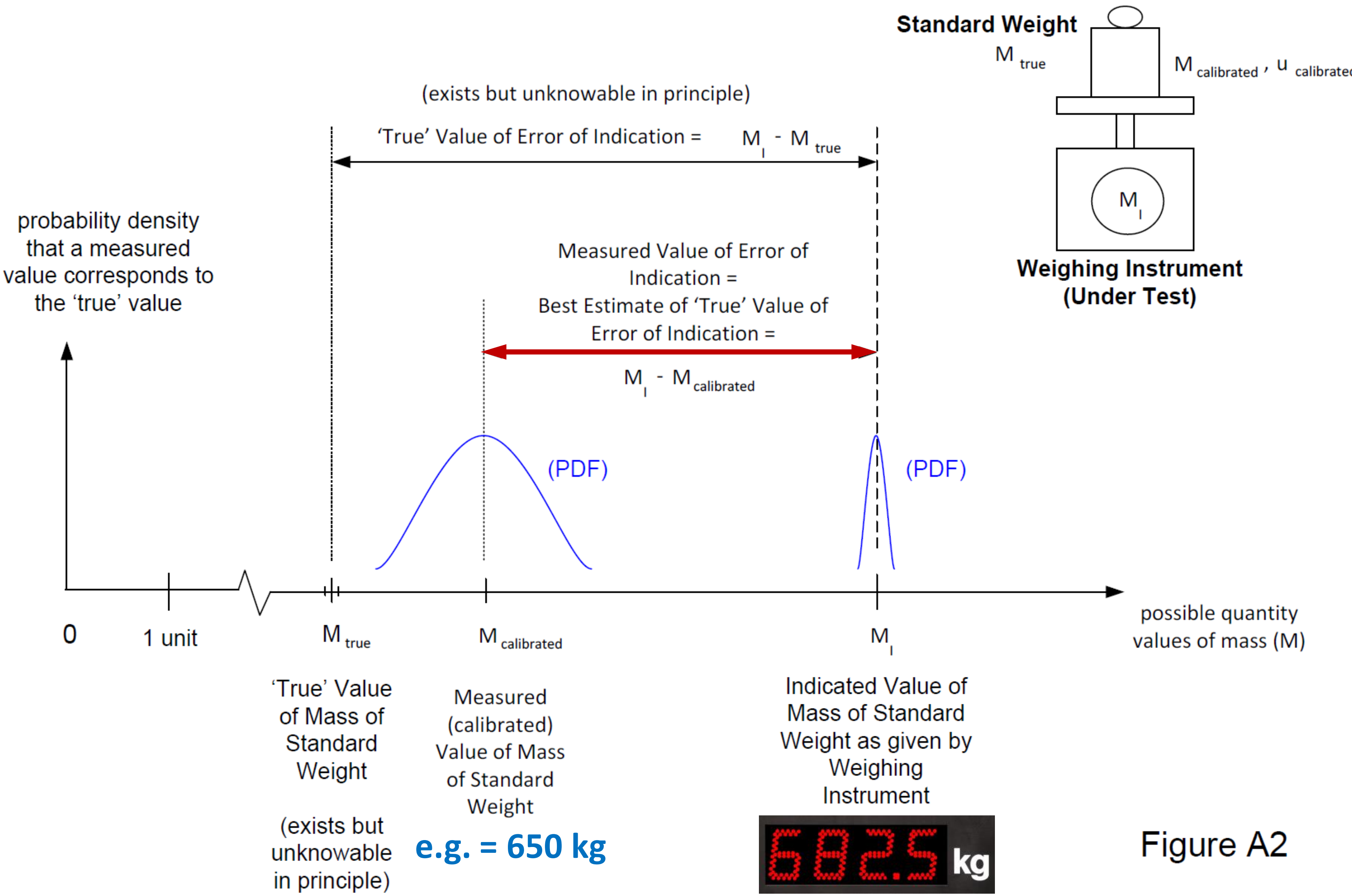
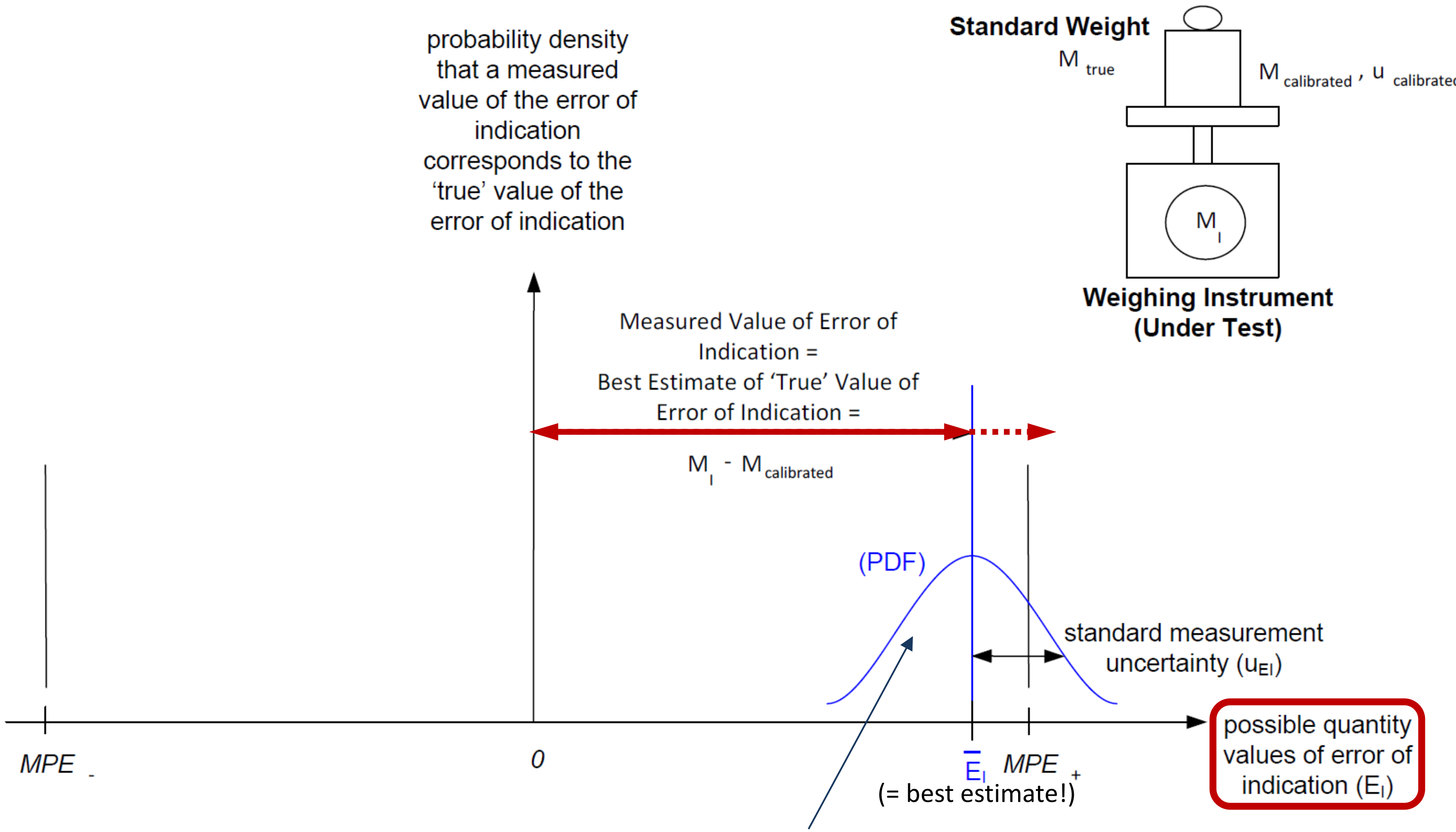


Figure A2

Error of Indication: Example for Weighing Instrument (Under Test)



This PDF is obtained by combining (sometimes called convolving) the two PDFs in Figure A2

Figure A3

How can “measurement error” and “measurement uncertainty” coexist when considering measurement in the context of verification?

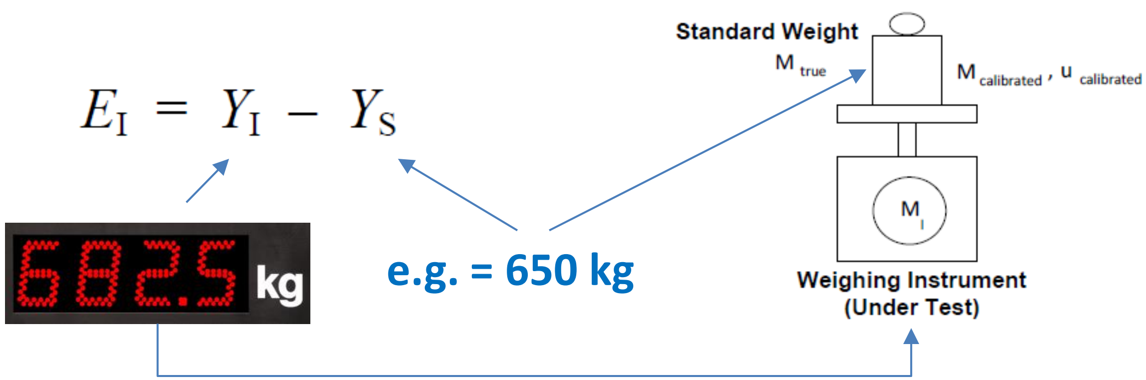
In fact, it makes sense to talk about the **uncertainty of a measured error of indication**!

The **measurement uncertainty** associated with the measurement standard(s) used when performing the verification test **must be taken into account** when making **(probabilistic) conformity assessment decisions**, since they contribute to the standard measurement uncertainty of the error of indication (u_{EI}).

4 Basic considerations

The **error of indication** is operationally taken to be the **difference** between **the indicated value (Y_I)** of the measuring instrument or system obtained when measuring the measurand, and **the value (Y_S)** of the same measurand as determined **when using a measurement standard:**

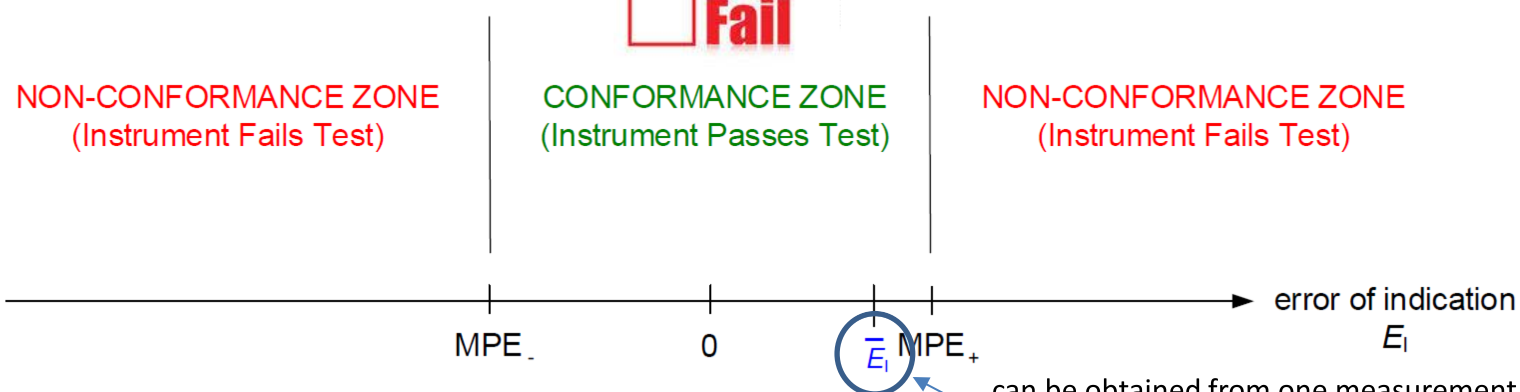
$$E_I = Y_I - Y_S$$



4 Basic considerations

Classical approach!

Using Error of Indication (E_I) and
Maximum Permissible Error (MPE)
for making a Conformity Decision
(Not Explicitly Incorporating Measurement Uncertainty)

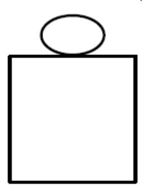


$\bar{E}_I$ can be obtained from one measurement or as the average of several measurements!

4 Basic considerations

For more complicated measurement standards (or systems), Y_s can be determined through use of a **‘measurement model’** that relates the value of the measurand to values (X_i) of ‘input quantities in a measurement model’ (that is, Y_s depends on, or is a function (f) of, the values X_i):

$$Y_S = f(X_1, X_2, \ldots X_n)$$



$M_{calibrated}$, $u_{calibrated}$

Standard Weight

e. g. air density (buoyancy!)
air temperature,
air pressure

during calibration of the weight

5 Conformity testing decisions that explicitly incorporates measurement uncertainty

Rather than being able to definitively state
that a measuring instrument **meets specified MPE requirements** and so passes a particular conformity test,

only a degree of belief
(or probability, expressed as a level of confidence)
can be stated that the measuring instrument conforms for each MPE requirement.

5 Conformity testing decisions that explicitly incorporate measurement uncertainty

Clause 5 focuses on the **explicit use of measurement uncertainty** for the purposes of **making conformity decisions**, such as when measurements are **performed in a laboratory environment**.

Clause 6 deals with measurements performed **outside a laboratory environment**!

ISO 17025

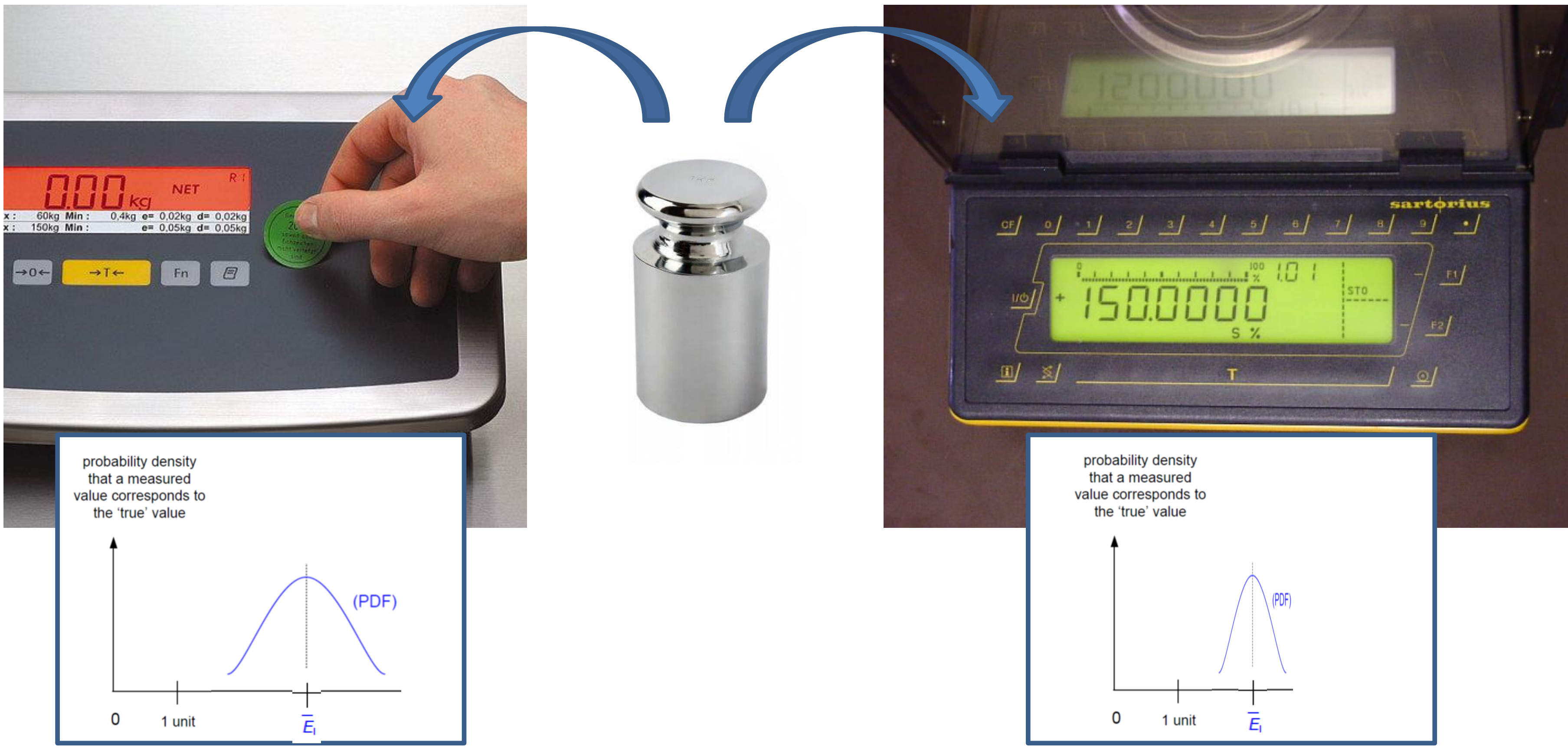
ISO/IEC 17025 states that

“[5.4.6.2] testing laboratories **shall have and shall apply procedures for estimating uncertainty of measurement**,”

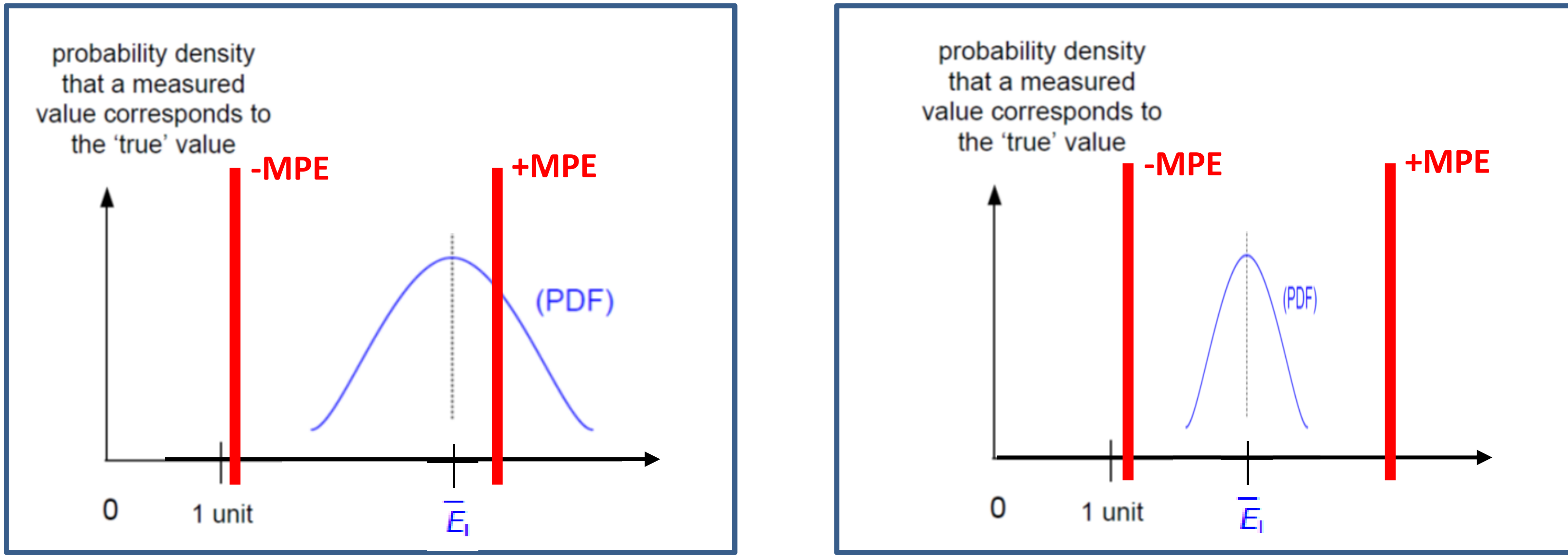
and, further

“[5.4.6.3] When estimating the uncertainty of measurement, **all uncertainty components** which are of importance in the given situation **shall be taken into account** using appropriate methods of analysis”.

Example: 2 balances, <u>different</u> resolution, <u>same</u> measurement standard

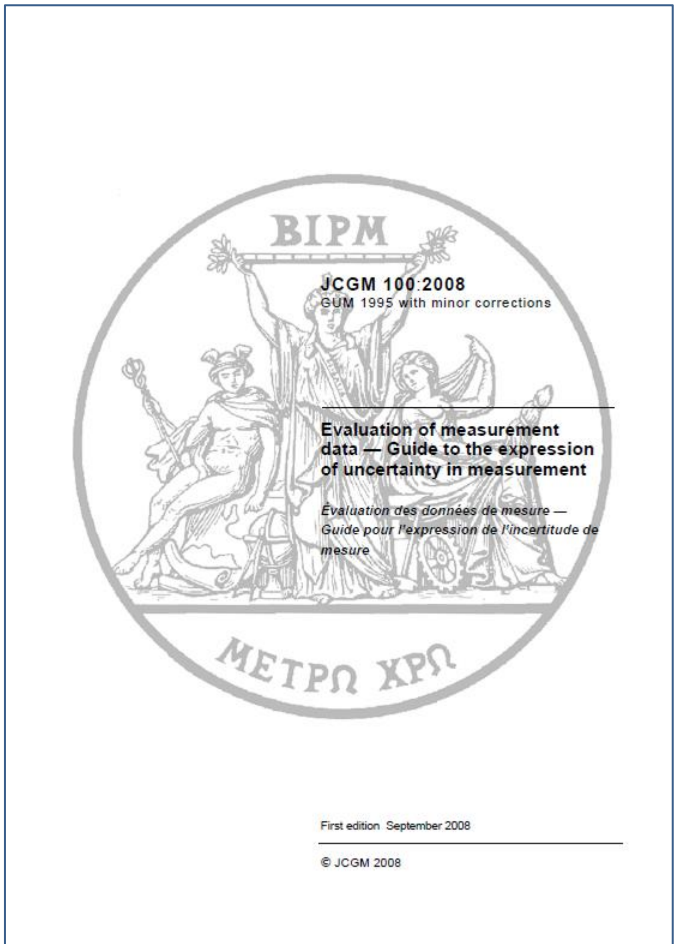


➡ Different measurement uncertainties!



➡ Different risks to make a wrong decision!

How to calculate measurement uncertainty?


BIPM

JCGM 100:2008
GUM 1995 with minor corrections

Evaluation of measurement data — Guide to the expression of uncertainty in measurement

Évaluation des données de mesure — Guide pour l'expression de l'incertitude de mesure

ΜΕΤΡΩ ΧΡΩ

First edition September 2008

© JCGM 2008

„GUM“ = Guide to the expression of Uncertainty in Measurement

Downloadable free of charge from:

https://www.bipm.org/utils/common/documents/jcgm/JCGM_100_2008_E.pdf

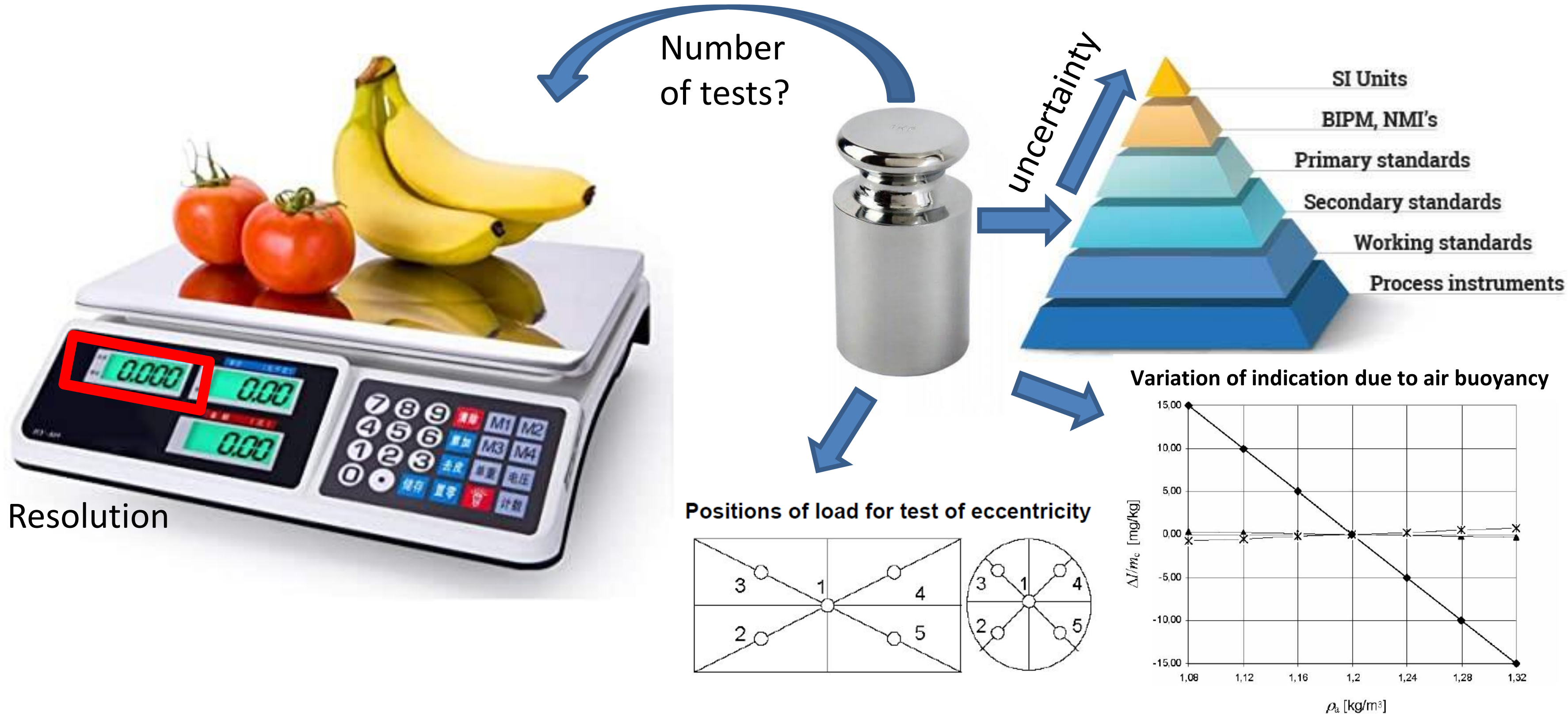
How to calculate measurement uncertainty?

OIML G 19, Chapter 8.2: Calculating measurement uncertainty

5 steps

(but of course one can also take directly the GUM!)

Step 1: „Describe the Instrument under Test (IuT) with the whole measuring system“



Step 1: „Describe the Instrument under Test (IuT) with the whole measuring system“

$$E_j = I_j - m_{\mathrm{ref}j}$$ ➡ e. g. see EURAMET cg18, chapter 6.2

(E_j = Error of indication, I_j = Indication, $m_{\mathrm{ref}j}$ = reference value of mass)

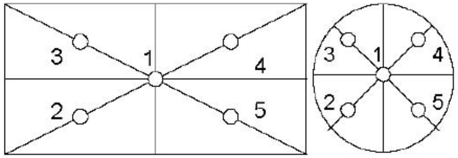
I_j

$m_{\mathrm{ref}j}$

Step 2: „Identify all the different kinds of tests, which are necessary“

For example:

Positions of load for test of eccentricity



$$I_j = I_L + \delta I_{\mathrm{dig}L} + \delta I_{\mathrm{rep}} + \delta I_{\mathrm{ecc}} - I_0 - \delta I_{\mathrm{dig}\,0} + \ldots$$ (cg18, chapter 7.1.1)

$$m_{\mathrm{ref}j} = m_{\mathrm{N}} + \delta m_{\mathrm{c}} + \delta m_{\mathrm{B}} + \delta m_D + \delta m_{\mathrm{conv}} + \delta m_{\ldots}$$ (cg18, chapter 7.1.2)

⇨ in this seminar we don‘t go into further details,
if you are interested into the details, please read cg18 or something else!

Step 3: „Calculate the associated measurement uncertainties“

$$u^2(I_j) = d_0^2/12 + d_I^2/12 + u^2(\delta I_{\mathrm{rep}}) + u_{\mathrm{rel}}^2(\delta I_{\mathrm{ecc}}) I^2$$

$$u^2(m_{\mathrm{ref}j}) = u^2(\delta m_{\mathrm{c}}) + u^2(\delta m_{\mathrm{B}}) + u^2(\delta m_D) + u^2(\delta m_{\mathrm{conv}})$$

Step 4: „Calculate a standard measurement uncertainty of the Error of Indication“

$$u^2(E_j) = u^2(I_j) + u^2(m_{\mathrm{ref}j})$$ = Variance!

Take the square root!

$$u(E_j)$$

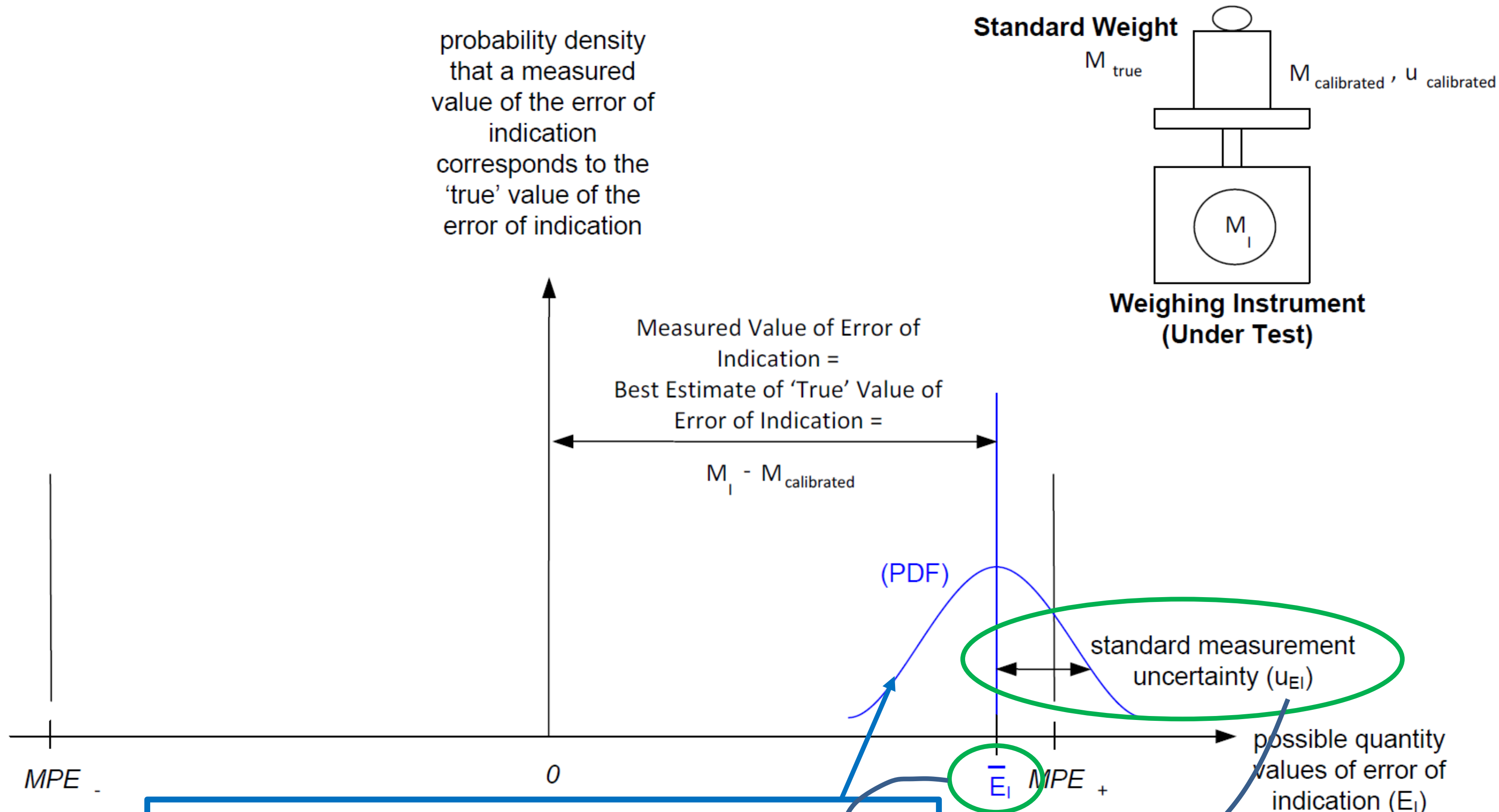
Step 5: „Calculate a expanded measurement uncertainty of the Error of Indication“

$$U\left(E_j\right) = k\,u\left(E_j\right)$$

The coverage factor k should be chosen such that the expanded uncertainty corresponds to a coverage probability of 95,45 %.

➡ **Fine, but what can we do with this result?**

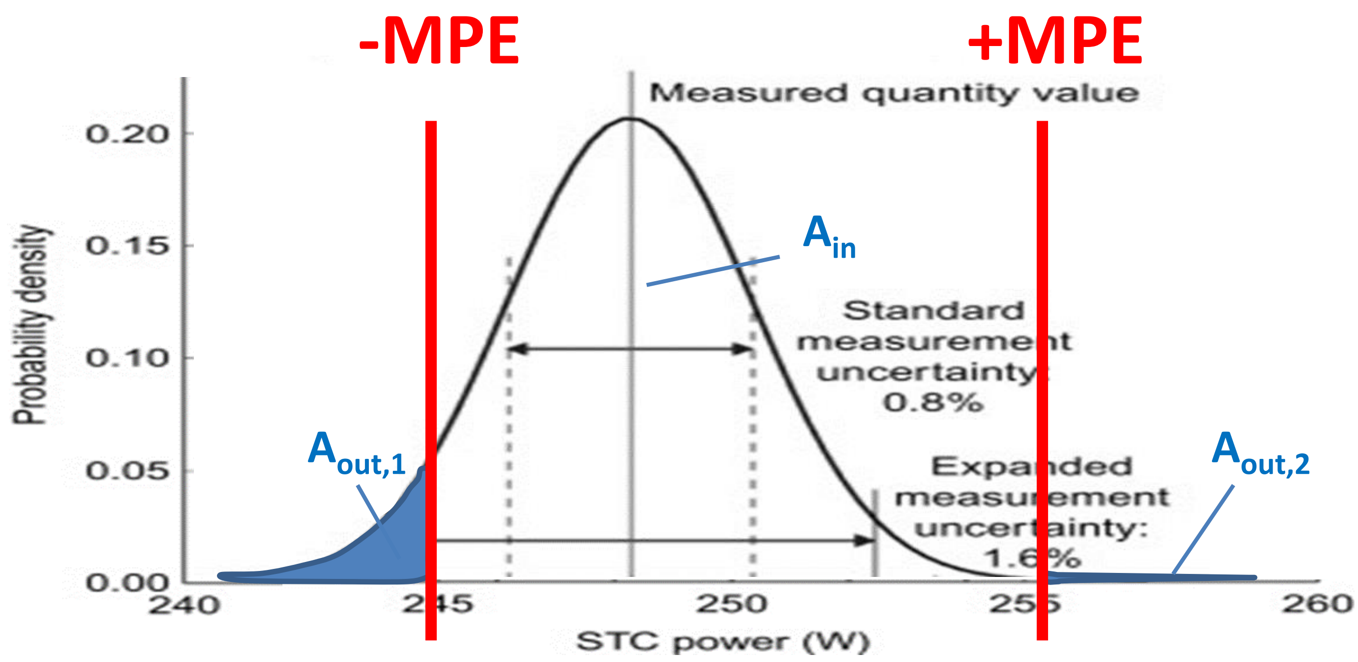
Error of Indication: Example for Weighing Instrument (Under Test)



Probability Density Function

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Now we can calculate our risk!



We “only“ need to calculate the areas under the curve!

Probability
Density
Function

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

integration

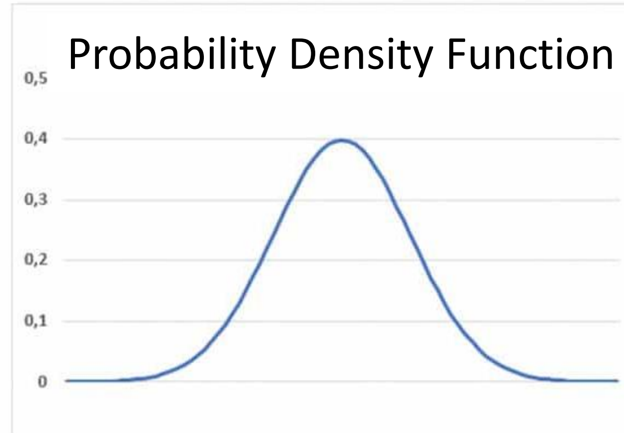
Distribution
Function

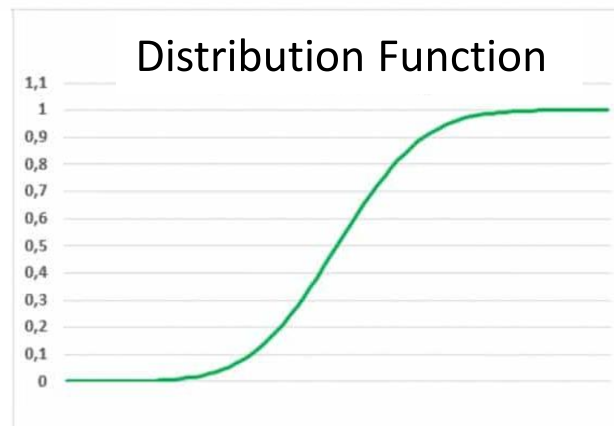
$$F(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}\, dt$$

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

integration

$$F(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}\, dt$$



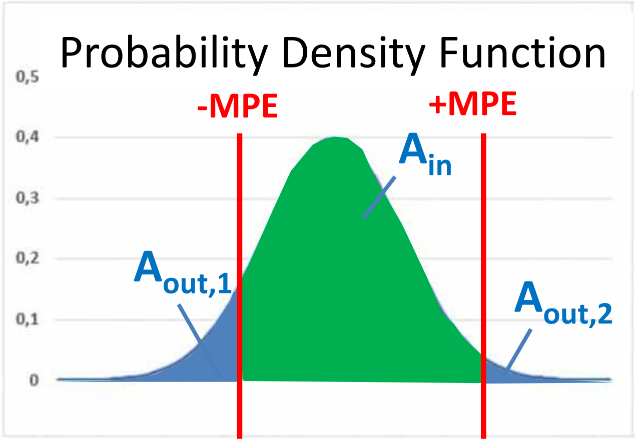


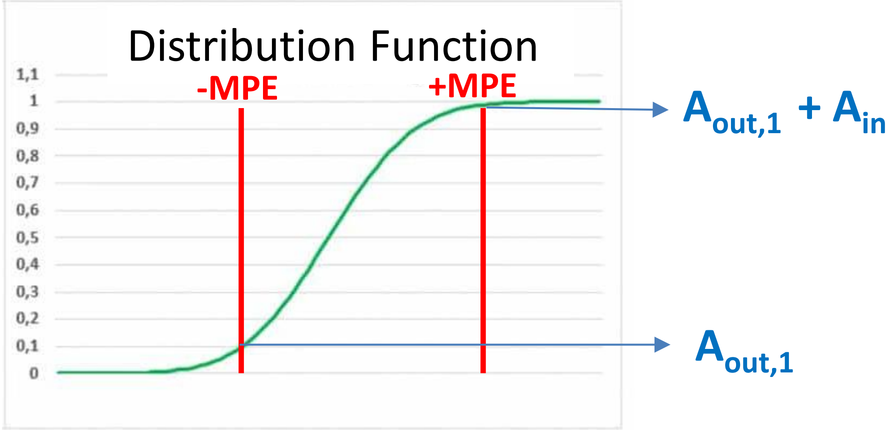
$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

integration

$$F(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}\, dt$$

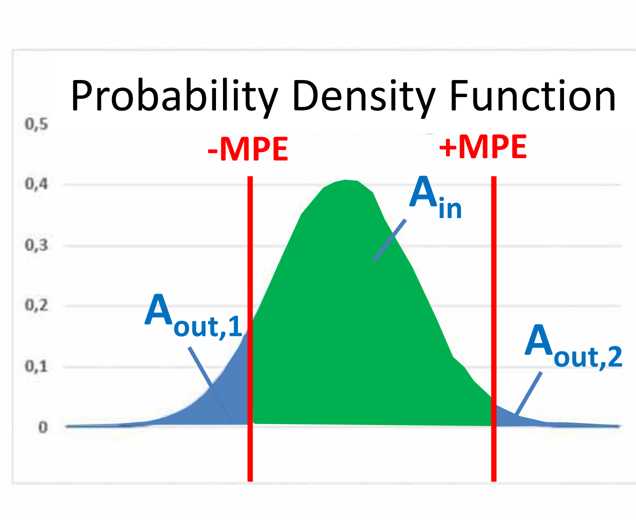
➨ Numerical integration necessary!

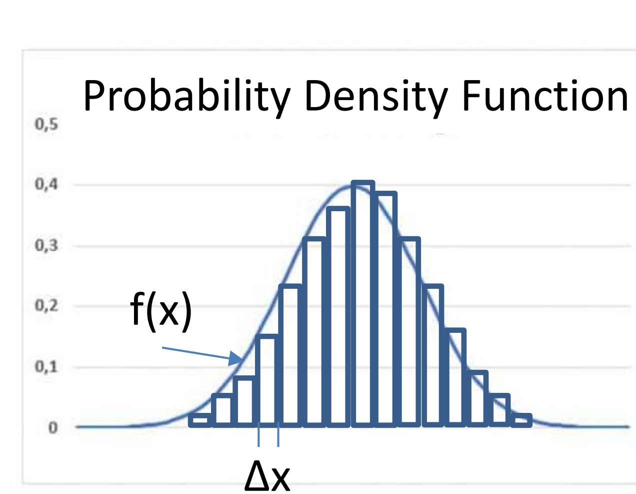




$$F(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \, dt$$

Numerical integration necessary!

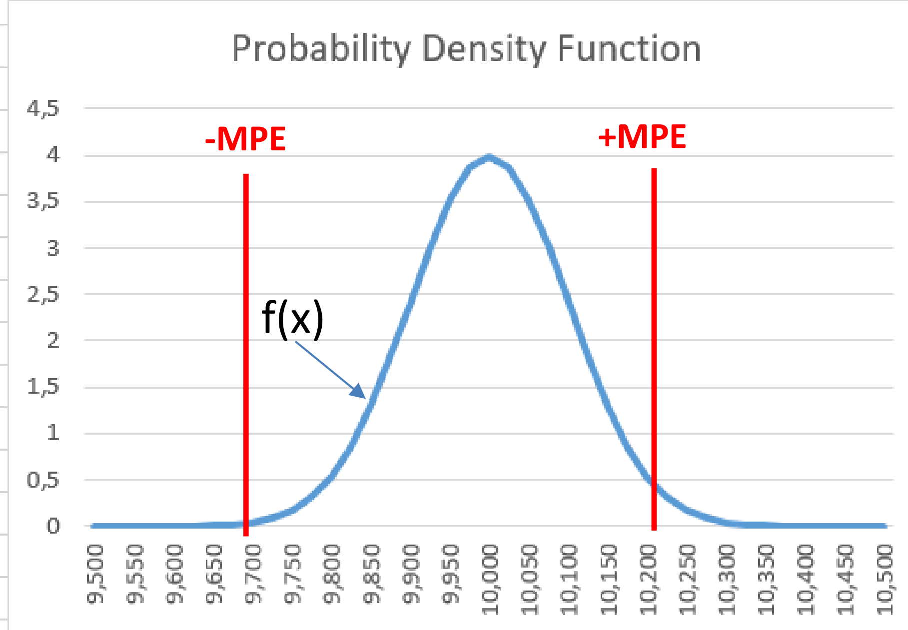


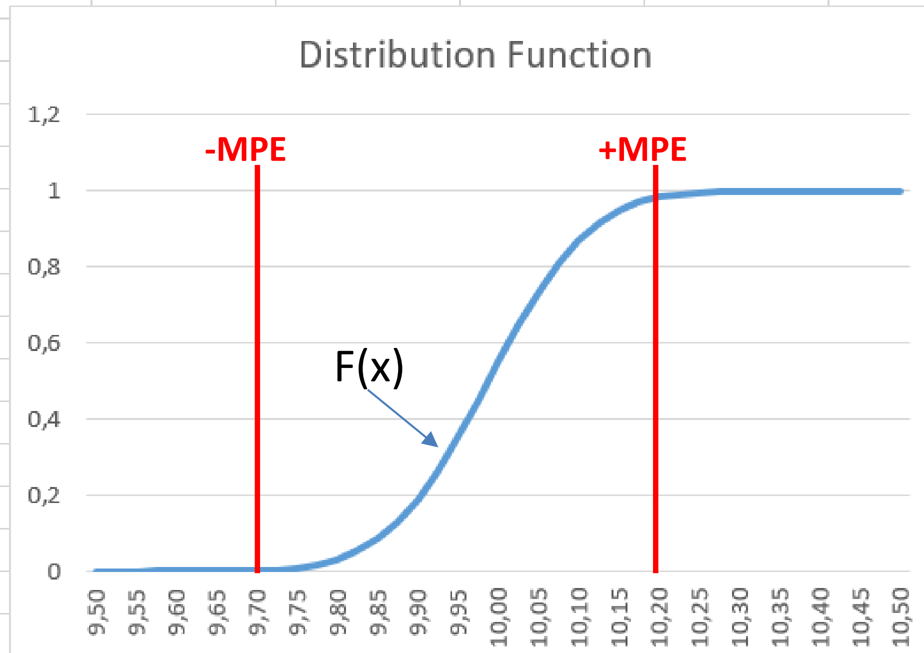


Simply a sum of rectangles!

	A	B	C	D
1	μ =	10		
2	σ =	0,1		
3	-MPE =	9,7		
4	+MPE =	10,2		
5	dt =	0,025		
6				
7	x-values:	f(x,σ,μ):	f(x,σ,μ) * dt:	F(x,σ,μ):
8	9,500	1,4867E-05	3,7168E-07	3,7168E-07
9	9,525	5,0295E-05	1,2574E-06	1,62906E-06
10	9,550	0,00015984	3,9959E-06	5,62499E-06
11	9,575	0,00047719	1,193E-05	1,75547E-05
12	9,600	0,0013383	3,3458E-05	5,10122E-05
13	9,625	0,00352596	8,8149E-05	0,000139161
14	9,650	0,00872683	0,00021817	0,000357332
15	9,675	0,02029048	0,00050726	0,000864594
16	9,700	0,04431848	0,00110796	0,001972556
17	9,725	0,09093563	0,00227339	0,004245947
18	9,750	0,175283	0,00438208	0,008628022
19	9,775	0,31739652	0,00793491	0,016562935
20	9,800	0,53990967	0,01349774	0,030060676
21	9,825	0,86277319	0,02156933	0,051630006
22	9,850	1,29517596	0,0323794	0,084009405
23	9,875	1,82649085	0,04566227	0,129671676
24	9,900	2,41970725	0,06049268	0,190164357
25	9,925	3,01137432	0,07528436	0,265448715
26	9,950	3,52065327	0,08801633	0,353465047
27	9,975	3,86668117	0,09666703	0,450132076
28	10,000	3,9894228	0,09973557	0,549867646
29	10,025	3,86668117	0,09666703	0,646534676
30	10,050	3,52065327	0,08801633	0,734551007
31	10,075	3,01137432	0,07528436	0,809835365
32	10,100	2,41970725	0,06049268	0,870328046

$$F(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}\, dt$$



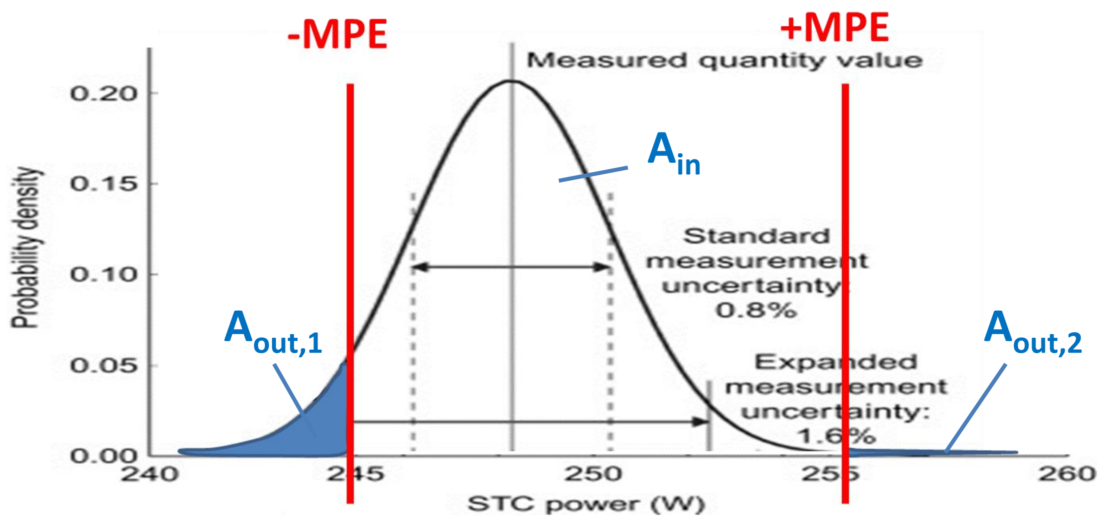


$$F(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}\, dt$$

Integration for 3 intervals of the example:

1. from $-\infty$ to -MPE = 0,001972 **= $A_{out,1}$**
2. from -MPE to +MPE = 0,983437 – 0,001972 = 0,981864 **= A_{in}**
3. from +MPE to $+\infty$ = 1,000000 – 0,983437 = 0,016563 **= $A_{out,2}$**

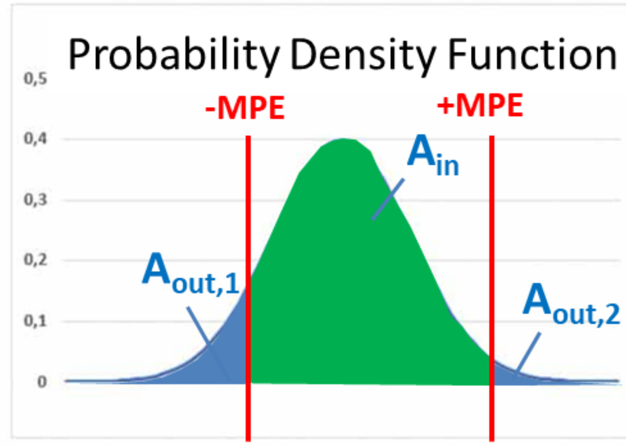
Risk calculation: „Instrument is conform“



Risk of a conform instrument being non-conform
resp. of a conform instrument failing the test:

$$\text{Risk} = \frac{A_{out,1} + A_{out,2}}{A_{out,1} + A_{out,2} + A_{in}} = \frac{A_{out}}{A_{total}}$$

$$F(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \, dt$$



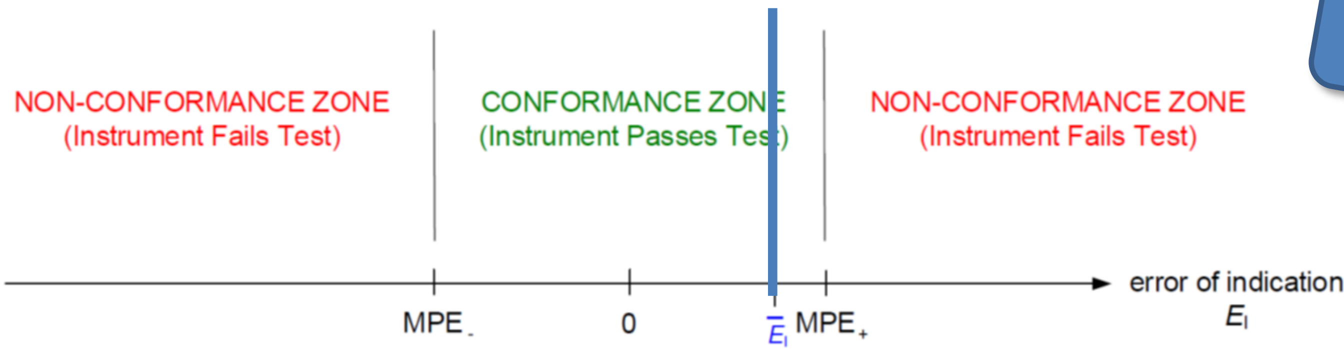
Integration for 3 intervals of the example:

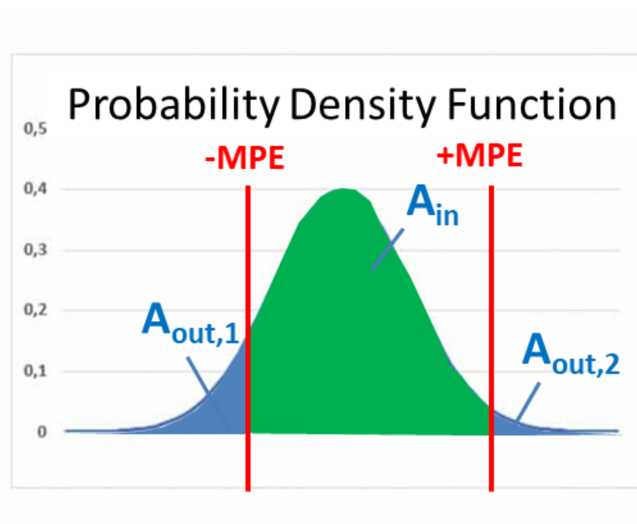
1. from $-\infty$ to -MPE = 0.001972 $= A_{out,1}$
2. from -MPE to +MPE = 0.983437 – 0.001972 = 0.981864 $= A_{in}$
3. from +MPE to $+\infty$ = 1.000000 – 0.983437 = 0.016563 $= A_{out,2}$

$$\text{Risk} = \frac{A_{out,1} + A_{out,2}}{A_{out,1} + A_{out,2} + A_{in}} = \frac{A_{out}}{A_{total}} = \frac{0.018536}{1.000000} = 1.85\ \%$$

Now you are able to calculate risks in conformity assessment using measurement uncertainties! (in principle!)

Classical approach!





Risk of a conform instrument being non-conform:

$$Risk = \frac{A_{out,1} + A_{out,2}}{A_{out,1} + A_{out,2} + A_{in}} = \frac{A_{out}}{A_{total}} = 1.85\ \%$$

Risk based approach!

Questions arise:

- Is a measuring instrument under test conform?

- What is acceptable? What is non-acceptable?

- What is the risk for passing the test, when it is non-conform?

- What is the risk for failing the test, when it is conform?

Questions?

Remarks?

Chapter 5.3
„Risks“ and „Decision rules“ associated with conformity decisions:

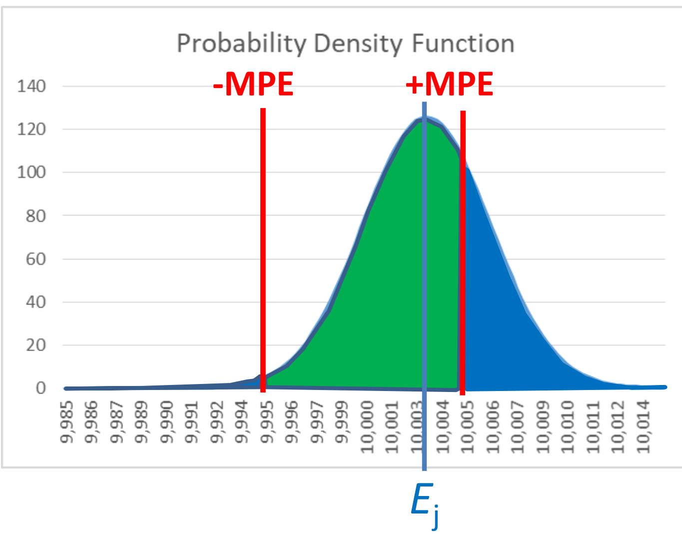
Three fundamental types of risks:

1) risk of false acceptance of a test,

2) risk of false rejection of a test result, and

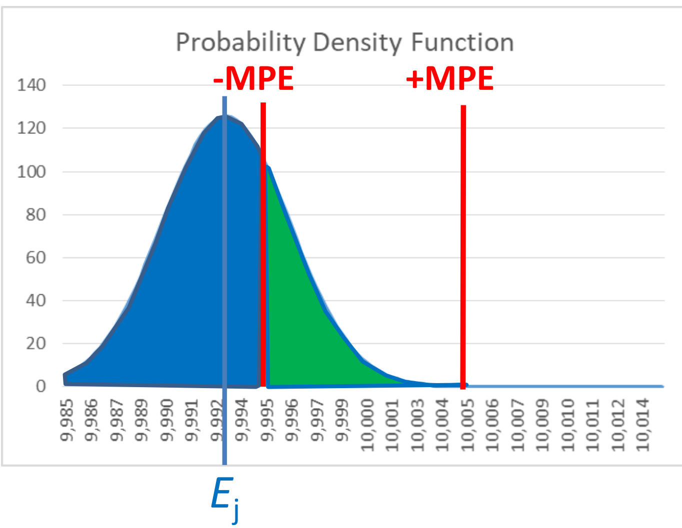
3) shared risk.

1) Risk of false acceptance:



- The measuring instrument is considered **to be conform**.
- But there is some area **outside** the MPEs under the curve!
- The risk is taken **by the evaluator**.
- Possible decision rule: „**Acceptance, if risk is lower than 5%**“

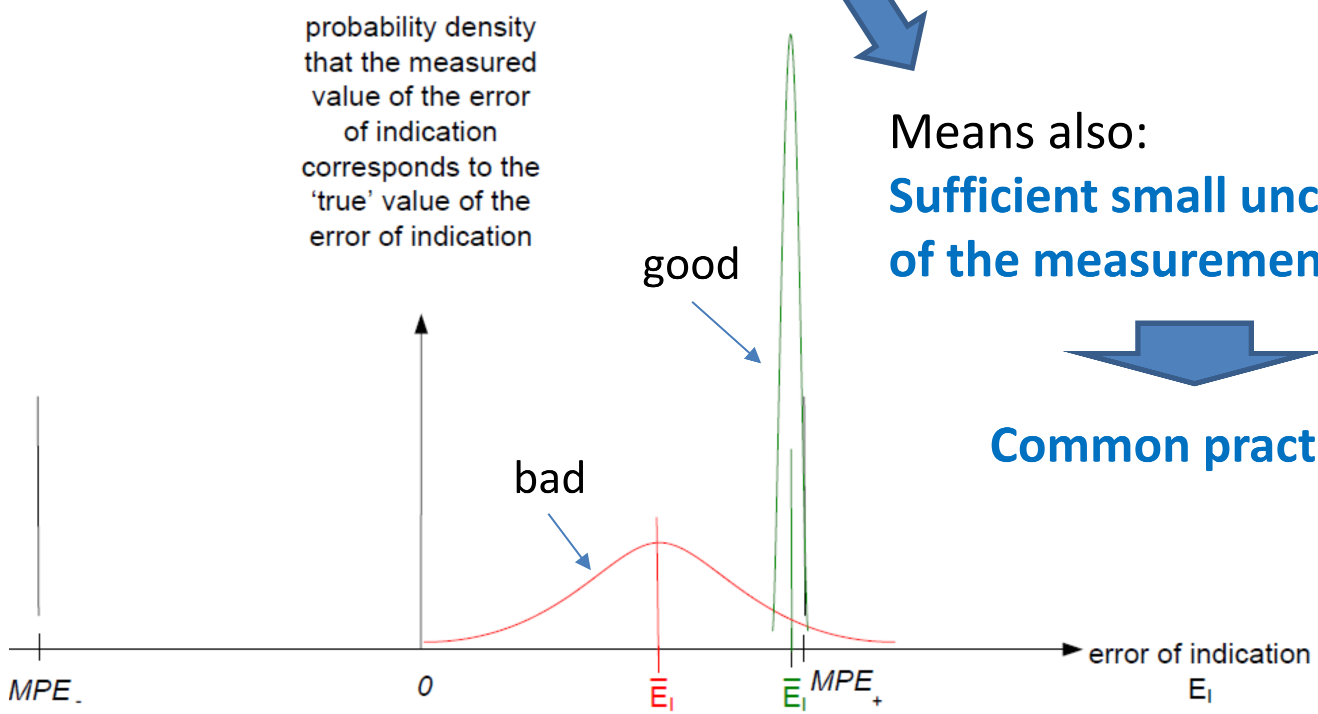
2) Risk of false rejection:



- The measuring instrument is considered **to be non-conform**.
- But there is some area **between** the MPEs under the curve!
- The risk is taken **by the manufacturer or seller**.
- **No decision rule for** a given test that incorporates **both risks** of false acceptance and risk of false rejection!

3) Shared risk:

- **No advantage or disadvantage** for the parties involved.
- Precondition: **sufficient small uncertainty** of the error of indication!



Means also:
Sufficient small uncertainty of the measurement standard!

Common practice!

3) Shared risk:

Means also:
Sufficient small uncertainty of the measurement standard! → **Common practice!**

Chapter 6:
Conformity testing decisions that do not explicitly incorporate measurement uncertainty

Measurements outside a lab:
No need to calculate measurement uncertainty for every measurement,
if
there is **always an underlying understanding** that the level of uncertainty in the measurement results **has been assured** and that the method of assurance **is well documented**.

	A	B	C	D
1	μ =	10		
2	σ =	0,1		
3	-MPE =	9,7		
4	+MPE =	10,2		
5	dt =	0,025		
6				
7	x-values:	f(x,σ,μ):	f(x,σ,μ) * dt:	F(x,σ,μ):
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11	9,575	0,00047719	1,193E-05	1,75547E-05
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19	9,775	0,31739652	0,00793491	0,016562935
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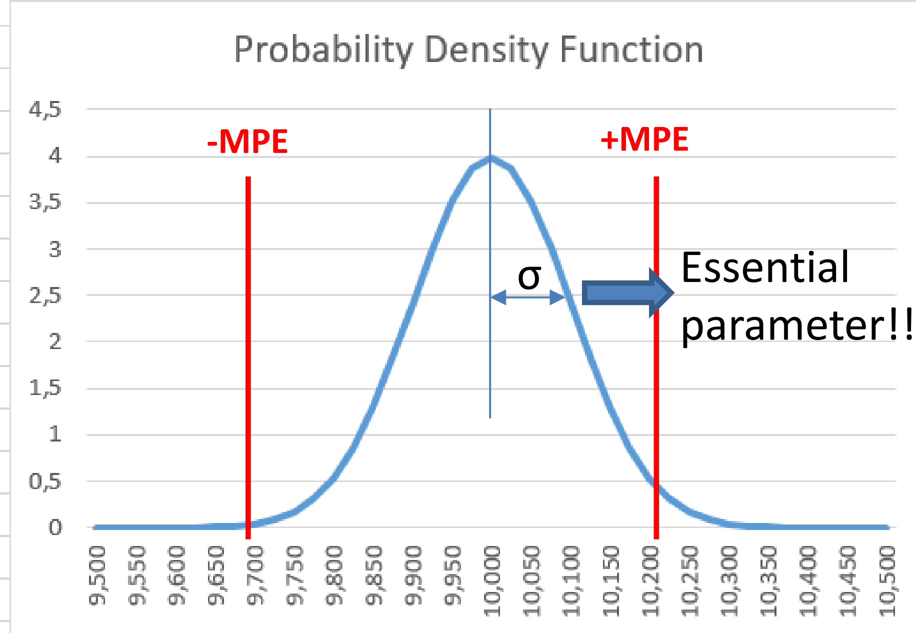
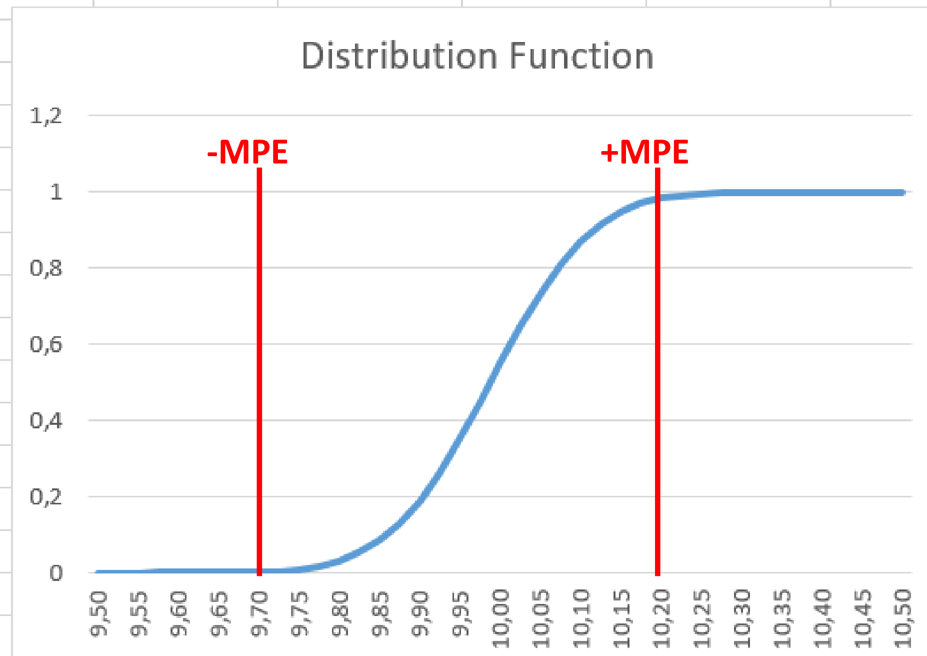
32	10,100	2,41970725	0,06049268	0,870328046

$$F(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt$$

As we need some underlying understanding, let's have a closer look!



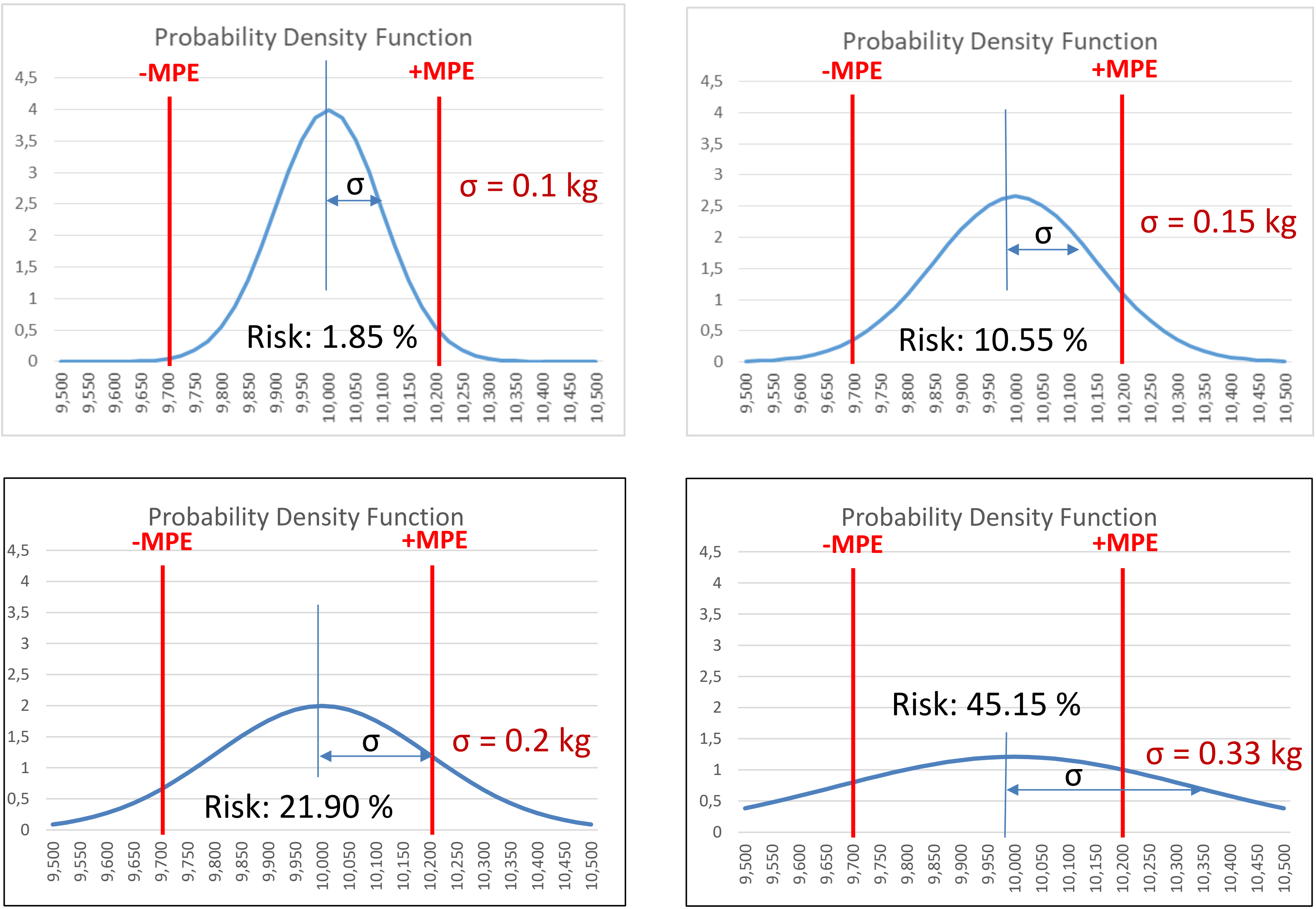


What, if the standard measurement uncertainty increases?

10 kg

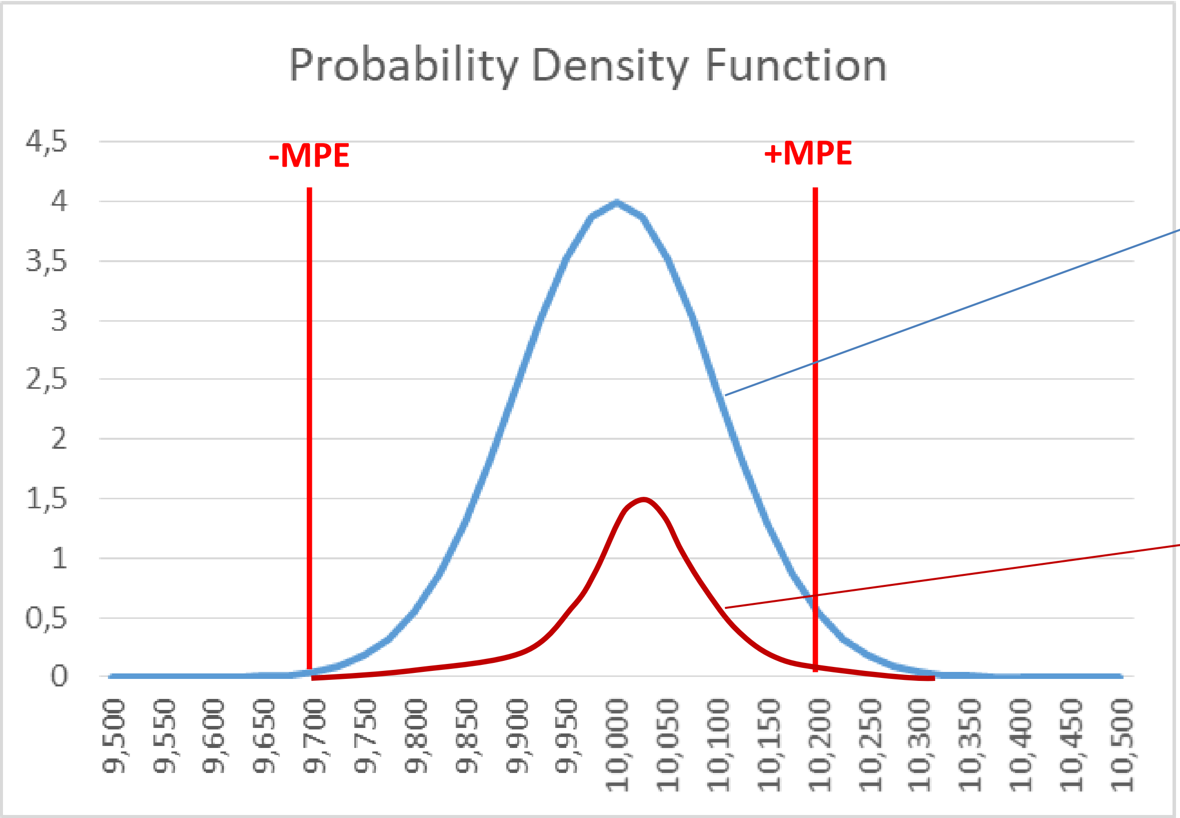
μ =	10	kg	
			Risk in %:
σ =	0,10	kg	1,85
σ =	0,15	kg	10,55
σ =	0,20	kg	21,90
σ =	0,25	kg	32,25
σ =	0,33	kg	45,15

A small increase of σ leads to a big change of the risk!



➡ Our conformity assessment procedure should result in a **narrow** probability density function!

Measurement uncertainty of the measurement standard as integral part



Standard measurement uncertainty

$$u^2(E_j) = u^2(I_j) + u^2(m_{\text{ref}j})$$

Contribution of the measurement standard

$$u^2(m_{\text{ref}j})$$

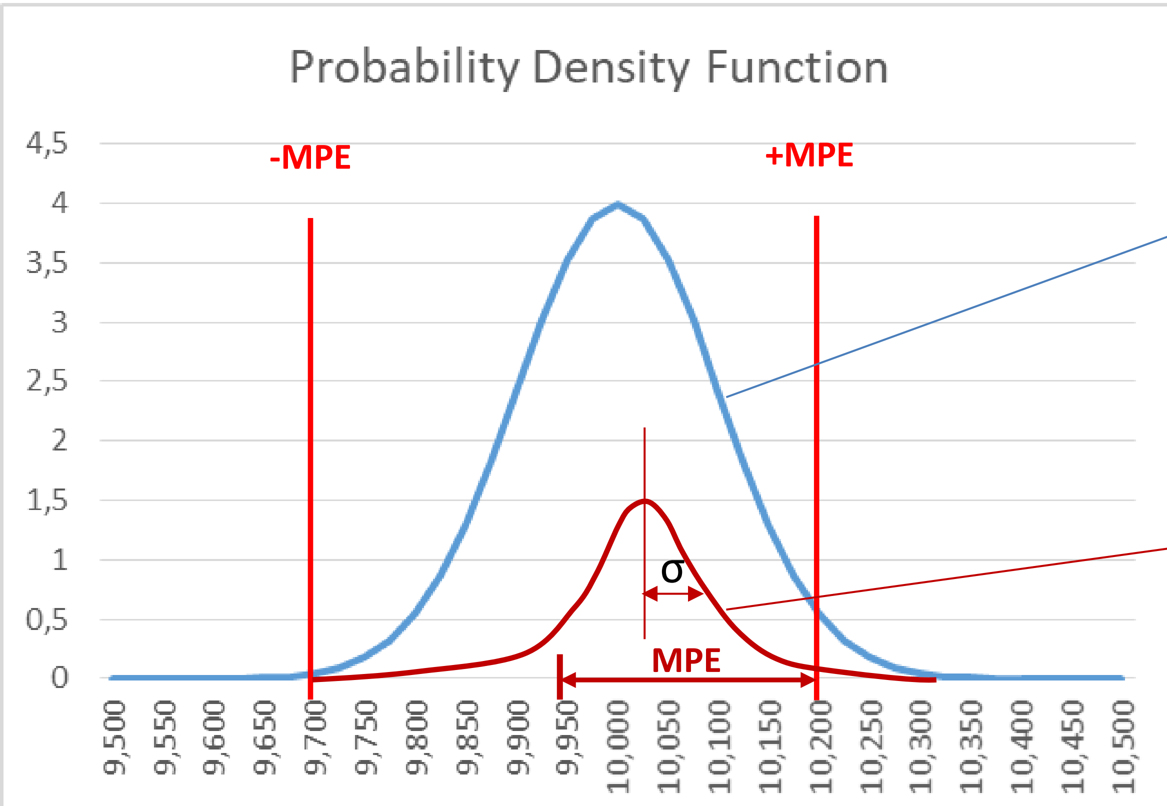
Should be „small enough"!

Practical question:

How big should
the measurement uncertainty of your
measurement standard (e. g. used for verification) be
compared with the maximum permissible error?

1/3 or 1/4 or 1/5 or 1/10 or….?

Measurement uncertainty of the measurement standard as integral part



Standard measurement uncertainty

$$u^2(E_j) = u^2(I_j) + u^2(m_{\mathrm{ref}j})$$

Contribution of the measurement standard

$$u^2(m_{\mathrm{ref}j})$$

➡ What should **the ratio** be between σ and MPE?

Practical example:
Verification of a balance

Non-automatic weighing instrument:

- Commercial balance
- Maximum load = 15 kg
- verification value = 5 g = maximum permissible error

Measurement standard:

- 10 kg weight
- according to OIML R111
- What class?

Practical example:

OIML R111:

10 kg

Table 1 Maximum permissible errors for weights (± δm in mg)

Nominal value*	Class E_1	Class E_2	Class F_1	Class F_2	Class M_1	Class M_{1-2}	Class M_2	Class M_{2-3}	Class M_3
10 kg	5.0	16	50	160	500		1 600		5 000

Discrepancy:

Maximum permissible error ⟺ Standard measurement uncertainty

How to calculate the standard measurement uncertainty σ from the maximum permissible error for the weight?

Physikalisch-
Technische
Bundesanstalt

DKD

„Guideline for the calibration of non-automatic weighing instruments“

Richtlinie DKD-R 7-2

Richtlinie zur Kalibrierung nichtselbsttätiger Waagen

Ausgabe 01/2018

https://doi.org/10.7795/550.20180928

Übersetzung des EURAMET Calibration Guide No. 18 Version 4.0 (11/2015)

https://www.ptb.de/cms/en/metrological-services/dkd/publications.html

COOMET

Chapter 7.1.2: Calculation of the standard measurement uncertainty of the reference weight

$$m_{\text{ref}} = m_{\text{N}} + \delta m_{\text{c}} + \delta m_{\text{B}} + \delta m_{D} + \delta m_{\text{conv}} + \delta m_{\dots} .$$

- m_{N}: 10 kg
- δm_{c}: Correction conventional weighing value
- δm_{B}: Buoyancy
- δm_{D}: Drift
- δm_{conv}: Convection effect (ΔT)

Practical example:

$$m_{\text{ref}} = m_{\text{N}} + \delta m_{\text{c}} + \delta m_{\text{B}} + \delta m_{D} + \delta m_{\text{conv}} + \delta m_{\dots}.$$

1. m_{N} = nominal value (in this example = 10kg)

2. In case of R111: $u(\delta m_{\text{c}}) = Tol/\sqrt{3}$ (e. g. Tol = 500 mg for M_1-weight)

3. $u_{\text{rel}}(\delta m_{\text{B}}) \approx (0{,}1\rho_0/\rho_{\text{c}} + mpe/(4m_{\text{N}}))/\sqrt{3}$ (ρ_0 = 1.2 kg/m³, ρ_{c} = 8000 kg/m³, *mpe* = 5 g in this example)

4. $u(\delta m_{D}) = D/\sqrt{3}$ (if there is no information about drift => D = *mpe*)

5. $u(\delta m_{\text{conv}}) = \Delta m_{\text{conv}}/\sqrt{3}$ (only relevant for class F_2 or higher! This value can be taken from a table in DKD-R 7.2))

Practical example:

Δm_{conv} in mg:

	ΔT in K							
m in kg	20	15	10	7	5	3	2	1
50	113,23	87,06	60,23	43,65	32,27	20,47	14,30	7,79
20	49,23	38,00	26,43	19,25	14,30	9,14	6,42	3,53
10	26,43	20,47	14,30	10,45	7,79	5,01	3,53	1,96
5	14,30	11,10	7,79	5,72	4,28	2,76	1,96	1,09
2	6,42	5,01	3,53	2,61	1,96	1,27	0,91	0,51
1	3,53	2,76	1,96	1,45	1,09	0,72	0,51	0,29
0,5	1,96	1,54	1,09	0,81	0,61	0,40	0,29	0,17
0,2	0,91	0,72	0,51	0,38	0,29	0,19	0,14	0,08
0,1	0,51	0,40	0,29	0,22	0,17	0,11	0,08	0,05
0,05	0,29	0,23	0,17	0,12	0,09	0,06	0,05	0,03
0,02	0,14	0,11	0,08	0,06	0,05	0,03	0,02	0,01
0,01	0,08	0,06	0,05	0,03	0,03	0,02	0,01	0,01

ΔT = Temperature difference between the temperature of the weight and the ambient air (positive difference = positive values and vice versa)

Practical example:

$$u^2(m_{\text{ref}}) = u^2(\delta m_{\text{c}}) + u^2(\delta m_{\text{B}}) + u^2(\delta m_D) + u^2(\delta m_{\text{conv}}) \ ,$$

	$u(\delta m_{\text{c}}) = Tol/\sqrt{3}$			Tol =	500	mg	
	=	288,68	mg				
	$u_{\text{rel}}(\delta m_{\text{B}}) \approx (0{,}1\rho_0/\rho_{\text{c}} + mpe/(4m_{\text{N}}))/\sqrt{3}$				rho_0 =	1,2	kg/m3
					rho_c =	8000	kg/m3
					mpe =	0,005	kg
					m_N =	10	kg
	=	8,0829E-05	(no unit! = relative measure => needs to be multiplied with 10 kg)				
	$u(\delta m_D) = D/\sqrt{3}$		D = mpe =	0,005	kg		
	=	2886,75	mg				
			Estimate:	from Table:			
	$u(\delta m_{\text{conv}}) = \Delta m_{\text{conv}}/\sqrt{3}$		DT = 5 K	Dm_conv =	7,79	mg	

Practical example:

$$u^2(m_{ref}) = u^2(\delta m_c) + u^2(\delta m_B) + u^2(\delta m_D) + u^2(\delta m_{conv}) \ ,$$

u =	288,68	0,00080829	2886,75	7,79
	mg	kg	mg	mg
u =	0,28867513	0,80829038	2,88675135	0,00779
	g	g	g	g
u^2 =	0,08333333	0,65333333	8,33333333	6,0684E-05
sum u^2 =	9,07006068			
u_mref =	**3,01**	**g**		

60 % of the MPE!!!

Result: The standard measurement uncertainty for the reference weight is relatively high compared with the MPE of 5 g!

Reason: high estimate for the drift! (without drift u_mref = 0.86 g)

What, if the standard measurement uncertainty increases?

Assumption:

For a moment we neglect the uncertainty of the indication!

The standard measurement uncertainty is **dominated by the** measurement uncertainty of the **measurement standard used** for the conformity assessment!

What is the contribution of the reference weight to the risk?

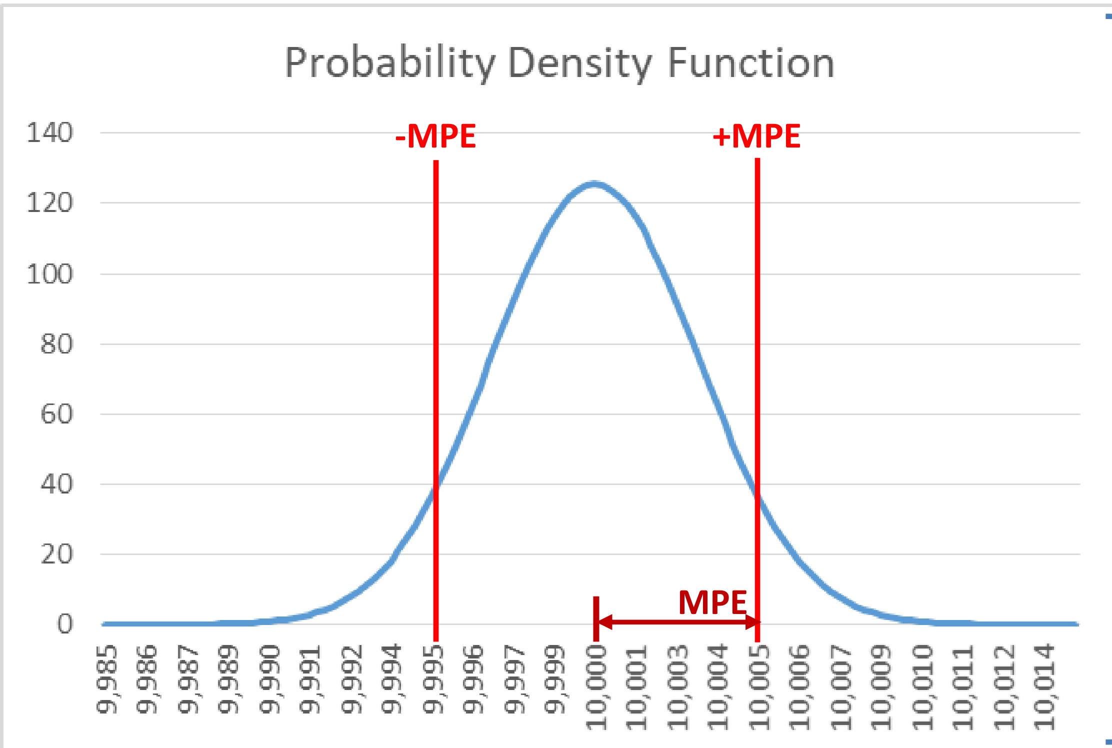
$$u^2(E) = \cancel{u^2(I)} + u^2(m_{\text{ref}})$$

(with $u^2(I)$ marked "= 0")

$$\cancel{u^2(I_j) = d_0^2/12 + d_I^2/12 + u^2(\delta I_{\text{rep}}) + u_{\text{rel}}^2(\delta I_{\text{ecc}}) I^2} \rightarrow = 0$$

$$u^2(m_{\text{ref}_j}) = u^2(\delta m_{\text{c}}) + u^2(\delta m_{\text{B}}) + u^2(\delta m_D) + u^2(\delta m_{\text{conv}})$$

Practical example:



Risk = 10.64 %
only from the reference weight = measurement standard!

With the assumption:

- No uncertainty contribution from indication!

Choice of reference weight in terms of risk:

OIML R111:

10 kg

Starting point:

Table 1 Maximum permissible errors for weights (± δm in mg)

Nominal value*	Class E_1	Class E_2	Class F_1	Class F_2	Class M_1	Class M_{1-2}	Class M_2	Class M_{2-3}	Class M_3
10 kg	5.0	16	50	160	500		1 600		5 000
$u(m_{ref})$	3.00 g	3.00 g	3.00 g	3.00 g	3.01 g		3.14 g		4.16 g
Risk:	10.53%	10.53%	10.53%	10.53%	10.64%		12.16%		24.20%

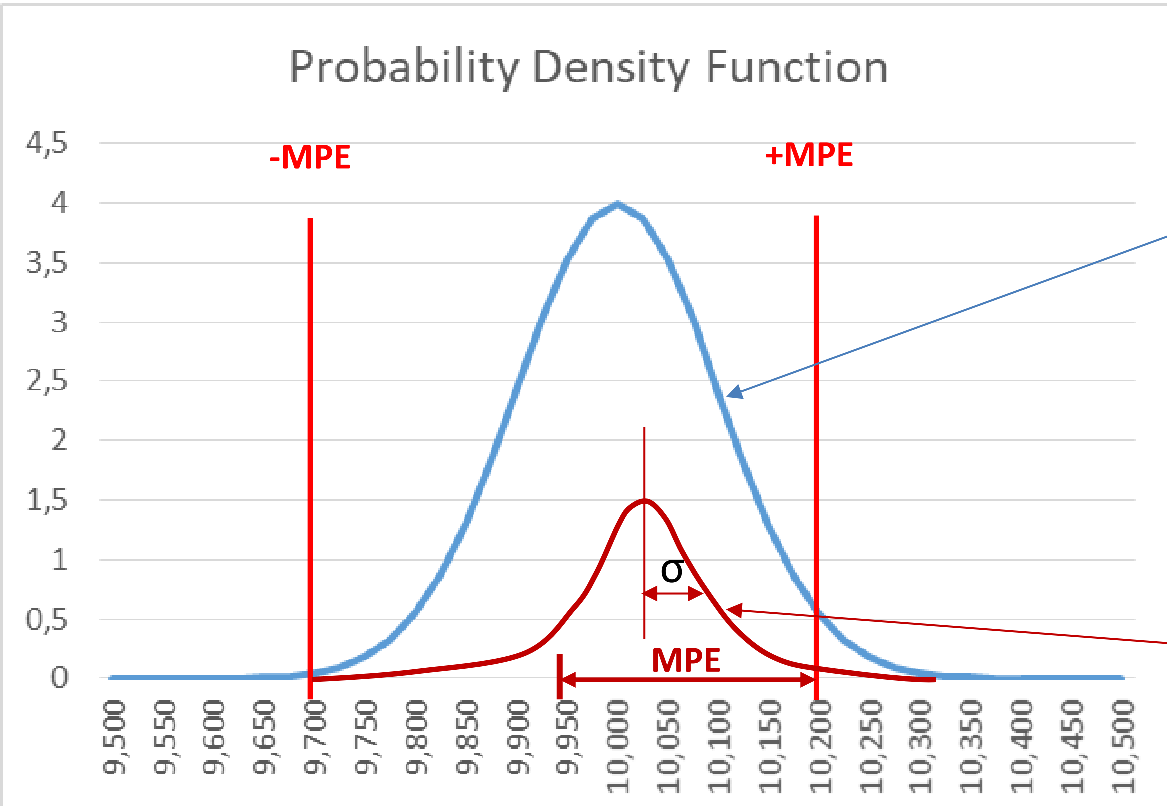
Higher quality = equal risk ←

→ Lower quality = higher risk

⇒ Contribution of the drift dominant!

⇒ **So far only a simplified approach has been taken!**
(Due to all those estimates taken by DKD 7.2
in combination OIML R111)

Finally, the try to draw a realistic picture:



Standard measurement uncertainty

$$u^2(E_j) = u^2(I_j) + u^2(m_{\mathrm{ref}j})$$

$u(m_{\mathrm{ref}})_{M_1}$

= Assumption!

Contribution of the measurement standard

$$u^2(m_{\mathrm{ref}j})$$

Finally, the try to draw a realistic picture:

$$u^2(m_{\text{ref}}) = u^2(\delta m_{\text{c}}) + u^2(\delta m_{\text{B}}) + u^2(\delta m_D) + u^2(\delta m_{\text{conv}}) \ ,$$

u =	288,68	0,00080829	2886,75	7,79
	mg	kg	mg	mg
u =	0,28867513	0,80829038	2,88675135	0,00779
	g	g	g	g
u^2 =	0,08333333	0,65333333	8,33333333	6,0684E-05
sum u^2 =	9,07006068			
u_mref =	**3,01**	**g**		

Buoyancy effects should be more investigated!

⇨ Assumption:
decreases by 10% with every better class
(starting with M1)

Finally, the try to draw a realistic picture:

$$u^2(m_{\text{ref}}) = u^2(\delta m_c) + u^2(\delta m_B) + u^2(\delta m_D) + u^2(\delta m_{\text{conv}}) \;,$$

u =	288,68	0,00080829	2886,75	7,79
	mg	kg	mg	mg
u =	0,28867513	0,80829038	2,88675135	0,00779
	g	g	g	g
u^2 =	0,08333333	0,65333333	8,33333333	6,0684E-05
sum u^2 =	9,07006068			
u_mref =	**3,01**	**g**		

Drift should also be investigated!

⇒ **Assumption:**
same magnitude as buoyancy

Contribution of reference weight to risk:

OIML R111:

10 kg

Table 1 Maximum permissible errors for weights (± δm in mg)

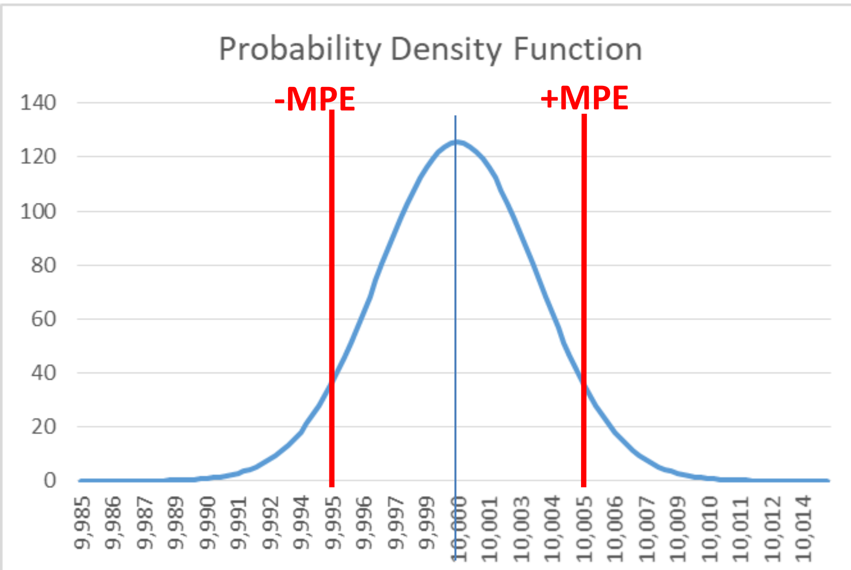
Nominal value*	Class E_1	Class E_2	Class F_1	Class F_2	Class M_1	Class M_{1-2}	Class M_2	Class M_{2-3}	Class M_3
10 kg	5.0	16	50	160	500		1 600		5 000

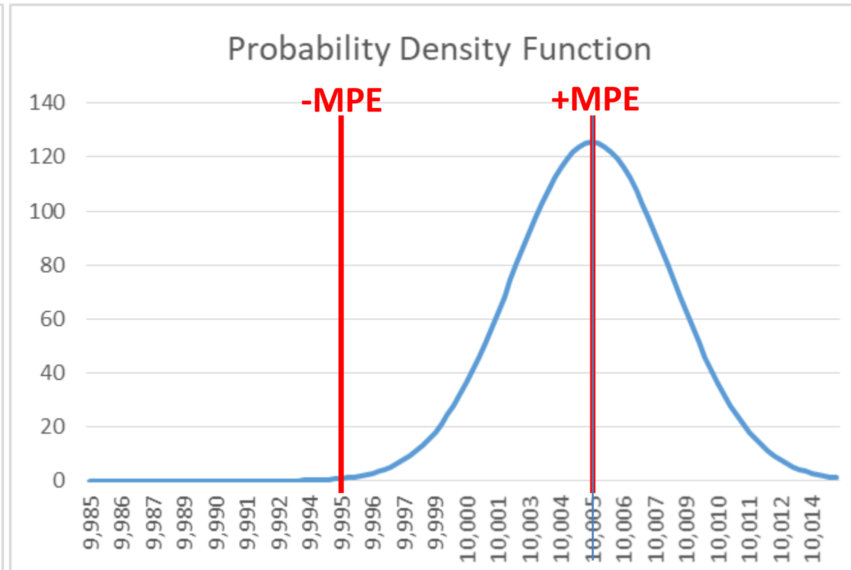
Starting point:

	Class E_1	Class E_2	Class F_1	Class F_2	Class M_1	Class M_2	Class M_3
$u(m_{ref})$	0.69 g	0.80 g	0.91 g	1.03 g	3.01 g	3.14 g	4.16 g
$u(I_j)$	3.01 g	3.01 g	3.01 g	3.01 g	3.01 g	3.01 g	3.01 g
$u(E_j)$	3.09 g	3.11 g	3.14 g	3.18 g	4.26 g	4.35 g	5.13 g
Risk:	11.57%	11.81%	12.16%	12.63%	25.32%	26.31%	34.25%
MPE:	5 g	5 g	5 g	5 g	5 g	5 g	5 g
$u(m_{ref})$ = % of MPE:	13.8%	16.0%	18.2%	20.6%	60.2%	62.8%	83.2%

Choice of the right standard for each conformity assessment task!

But be aware of the location of the error of indication E_j !





E_j E_j

➡ Risk = 12.63 % ➡ Risk ≈ 50 %

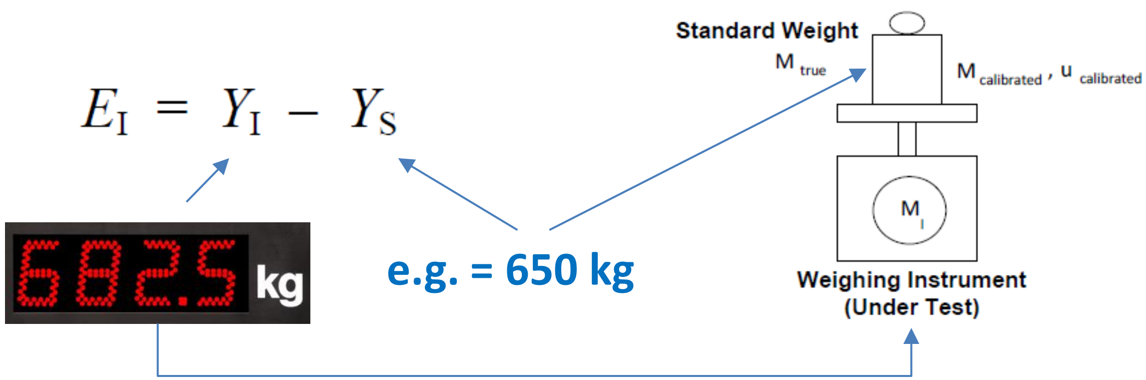
… with the same (good) reference weight!

Conclusion no. 1:

Classical approach!

Risk based approach!

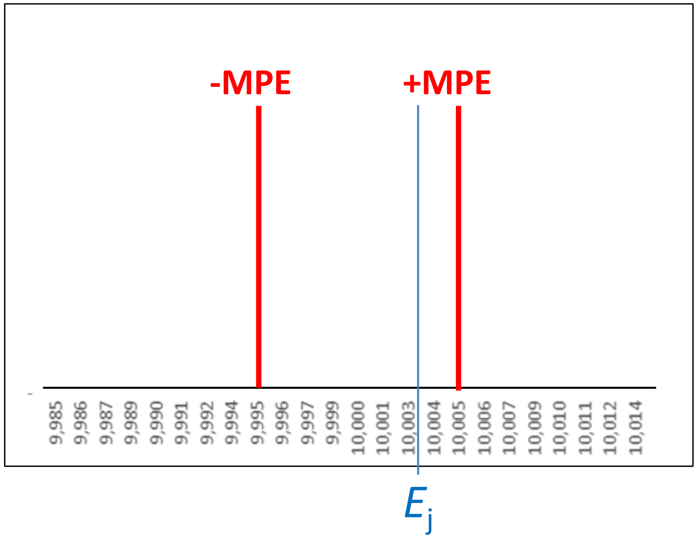
$$E_{\mathrm{I}} = Y_{\mathrm{I}} - Y_{\mathrm{S}}$$



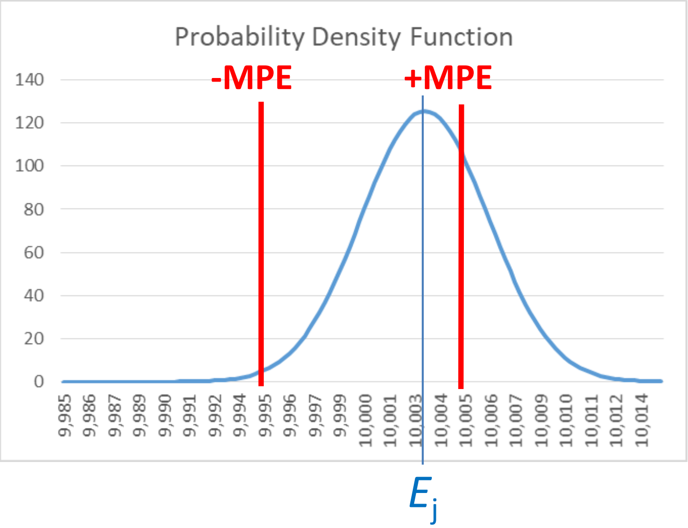
We have to measure the Error of Indication E_{I},
i. e. we have to apply a measurement standard and to read the indication from the measuring instrument.

Conclusion no. 2:

Classical approach!



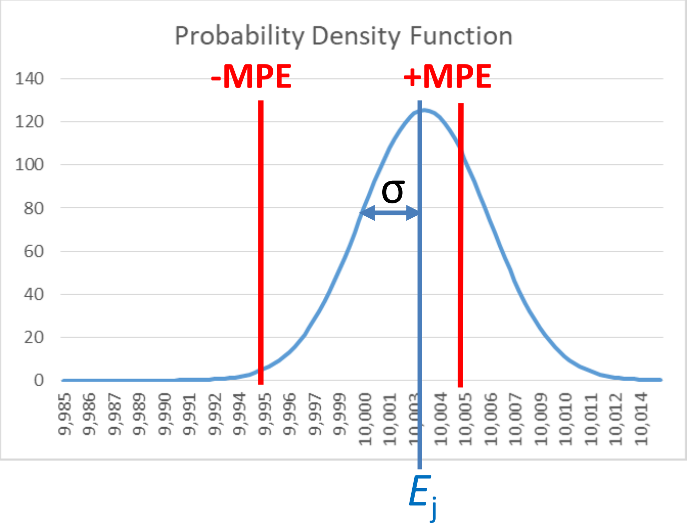
Risk based approach!



The error of indication should be within the interval between –MPE and +MPE in both cases to decide the measuring instrument is conform.

Conclusion no. 3:

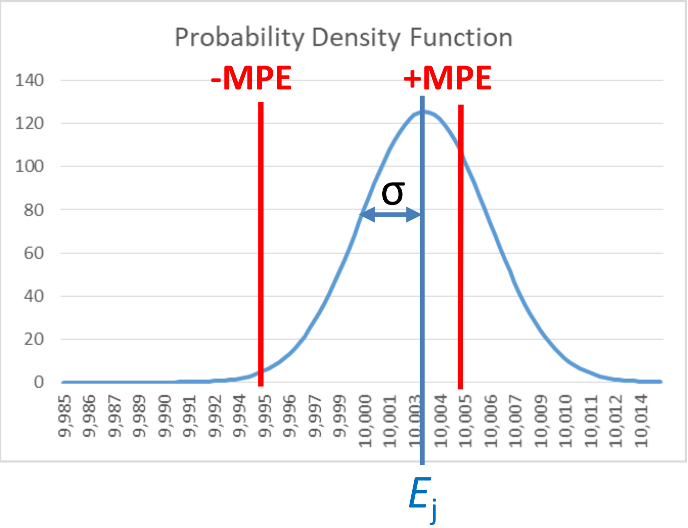
Risk based approach!



We have to calculate the standard uncertainty σ of the Error of Indication E_I ,
as the sqareroot of the sum of the variances of the uncertainties of indication and the measurement standard.

Conclusion no. 4:

Risk based approach!



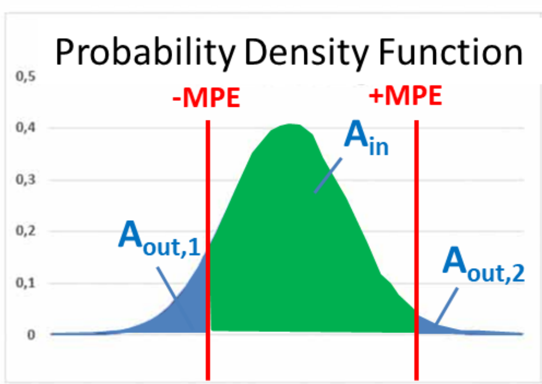
By applying integration in respect to the MPEs, we can calculate the area under the Probability Density Function to determine the areas inside and outside the MPEs and therefore the risk.

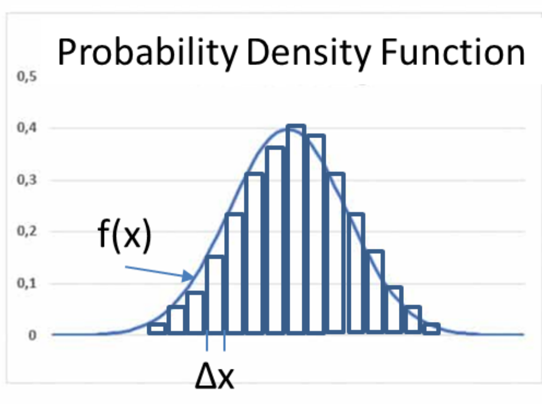
Conclusion no. 5:

Risk based approach!

$$F(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt$$

Numerical integration necessary!

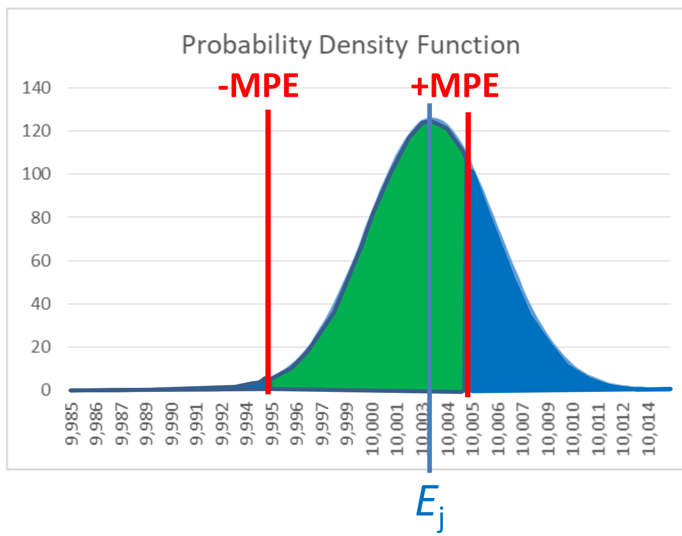




Simply a sum of rectangles!

Conclusion no. 6:

Risk based approach!

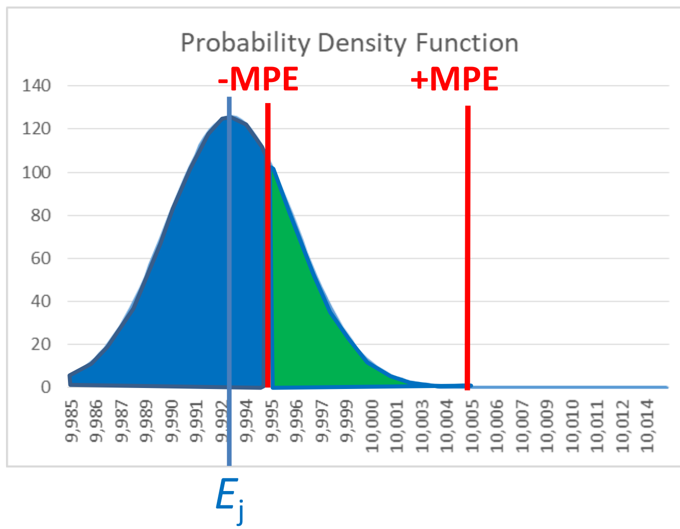


There is a risk of a **conform instrument being non-conform** resp. of a conform instrument failing the test:

$$\text{Risk} = \frac{A_{out}}{A_{total}}$$

Conclusion no. 7:

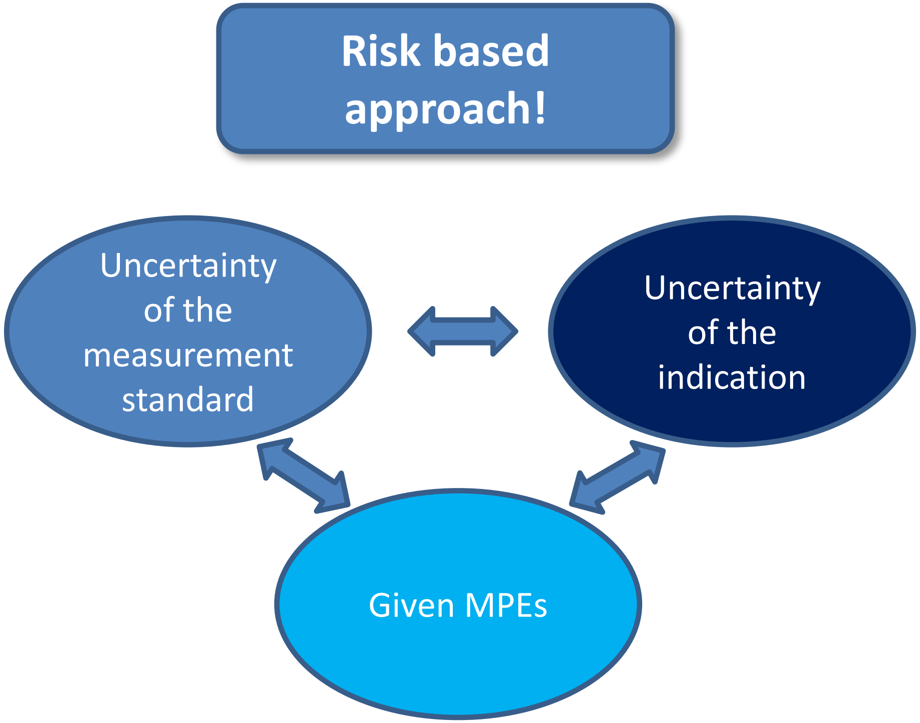
Risk based approach!



There is a risk of a **non-conform instrument being conform** resp. of a non-conform instrument passing the test:

$$\Rightarrow \quad \text{Risk} = \frac{A_{in}}{A_{total}}$$

Conclusion no. 8:



➡ **We have to balance the uncertainty of the measurement standard with the uncertainty of the indication and the given MPEs to make sound conformity assessment decisions,** for example uncertainty of the meas. standard = 1/5 of MPE.

Final conclusion:

Risk based approach!

Whatever we do in conformity assessment, we have to believe and we can‘t be 100 % sure. There is a risk …

Questions?

Remarks?

Software:

Software:

I have no contract with any of the manufacturers of the products, which will be introduced!

I didn‘t get any of the commercial software free of charge or any gifts etc.

All the information is just my personal, independent opinion!

Articles from the internet:

2004 Measurement Science Conference Anaheim, CA

A Comprehensive Comparison of Uncertainty Analysis Tools

Suzanne Castrup
Integrated Sciences Group
14608 Casitas Canyon Rd.
Bakersfield, CA 93306
661-872-1683
scastrup@isgmax.com

ABSTRACT

In recent years, compliance with ISO/IEC 17025, as well as other testing and calibration standards, has elevated the importance of estimating and reporting measurement uncertainty. ISO/TAG4/WG3 (the GUM) and ANSI/NCSL Z540-2-1997 [1] (the U.S. version of the GUM) provide guidelines for conducting an uncertainty analysis. Unfortunately, the implementation of these guidelines can be a daunting task, especially if one is not conversant in the necessary mathematical and statistical concepts. Consequently, testing and calibration personnel must often find off-the-shelf tools that meet their analysis requirements.

This paper presents a review and comparison of a number of software applications, both commercial and freeware, that have been developed in the past several years. Methodology, functionality, user-friendliness, documentation, technical support and other key criteria are addressed. Recommended guidelines for selecting an uncertainty analysis tool are also provided.

Articles from the internet:

Accred Qual Assur (2005) 10: 373–381
DOI 10.1007/s00769-005-0005-8

PRACTITIONER'S REPORT

J. M. Jurado
A. Alcázar

A software package comparison for uncertainty measurement estimation according to GUM

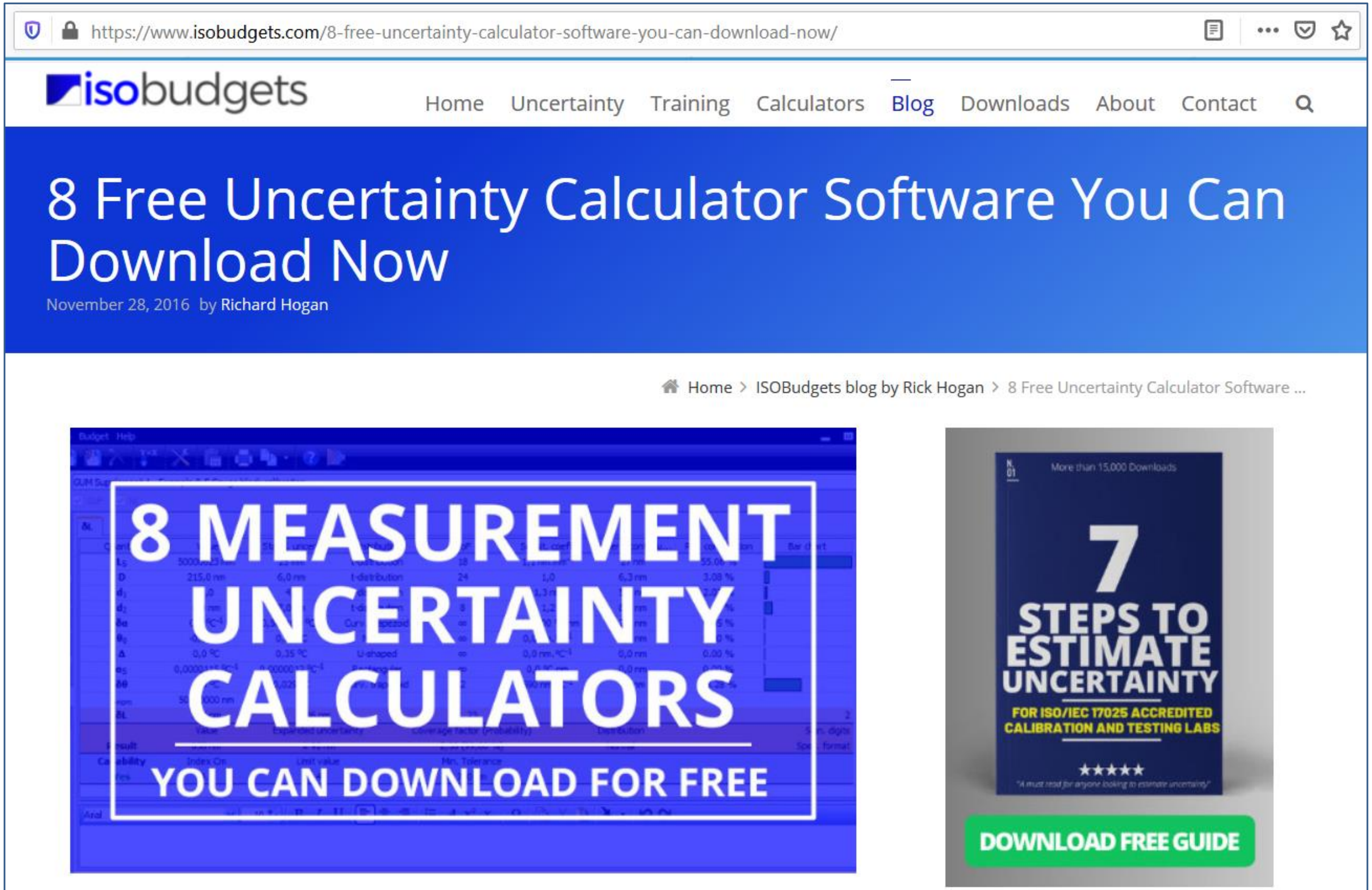
Received: 21 March 2005
Accepted: 9 June 2005
Published online: 2 September 2005

J. M. Jurado (✉) · A. Alcázar
Department of Analytical Chemistry,
University of Seville, Spain,
e-mail: jmjurado@us.es

Abstract Six commercial programs devoted to the estimation of measurement uncertainty were compared for feasibility in order to be applied in routine chemical analysis. The main features of each program were discussed. They were applied to two well-documented case studies. Several screen captures were considered for illustration.

Keywords Measurement uncertainty · GUM approach · Monte-Carlo simulation

Articles from the internet:



Commercial software:

- **@Risk,** palisade (USA)
  www.palisade.com

- **Crystal Ball**, Oracle (USA)
  www.oracle.com/applications/crystalball/

- **GUMsim**, quo data GmbH (Germany)
  www.quodata.de

Commercial software:

- **GUM workbench,** Metrodata GmbH (Germany)
  www.metrodata.de

- **QMSys GUM Standard**, Qualisyst Ltd. (Bulgaria)
  http://www.qsyst.com/qualisyst_en.htm

- **Uncertainty Sidekick Pro**, Integrated Sciences Group (USA)
  www.isgmax.com

- **Uncertainty Toolbox**, Quametec (USA)
  www.qimtonline.com

@Risk:

Overview

@RISK (pronounced "at risk") is an add-in to Microsoft Excel that lets you analyze risk using Monte Carlo simulation. @RISK shows you virtually all possible outcomes for any situation—and tells you how likely they are to occur. This means you can judge which risks to take on and which ones to avoid—critical insight in today's uncertain world.

- Works with Microsoft Excel
- Avoid Pitfalls and Uncover Opportunities with Risk Analysis
- Plan Better Strategies
- Identify Factors Causing Risk and Protect Yourself with Contingency Planning
- Communicate Risk To Others

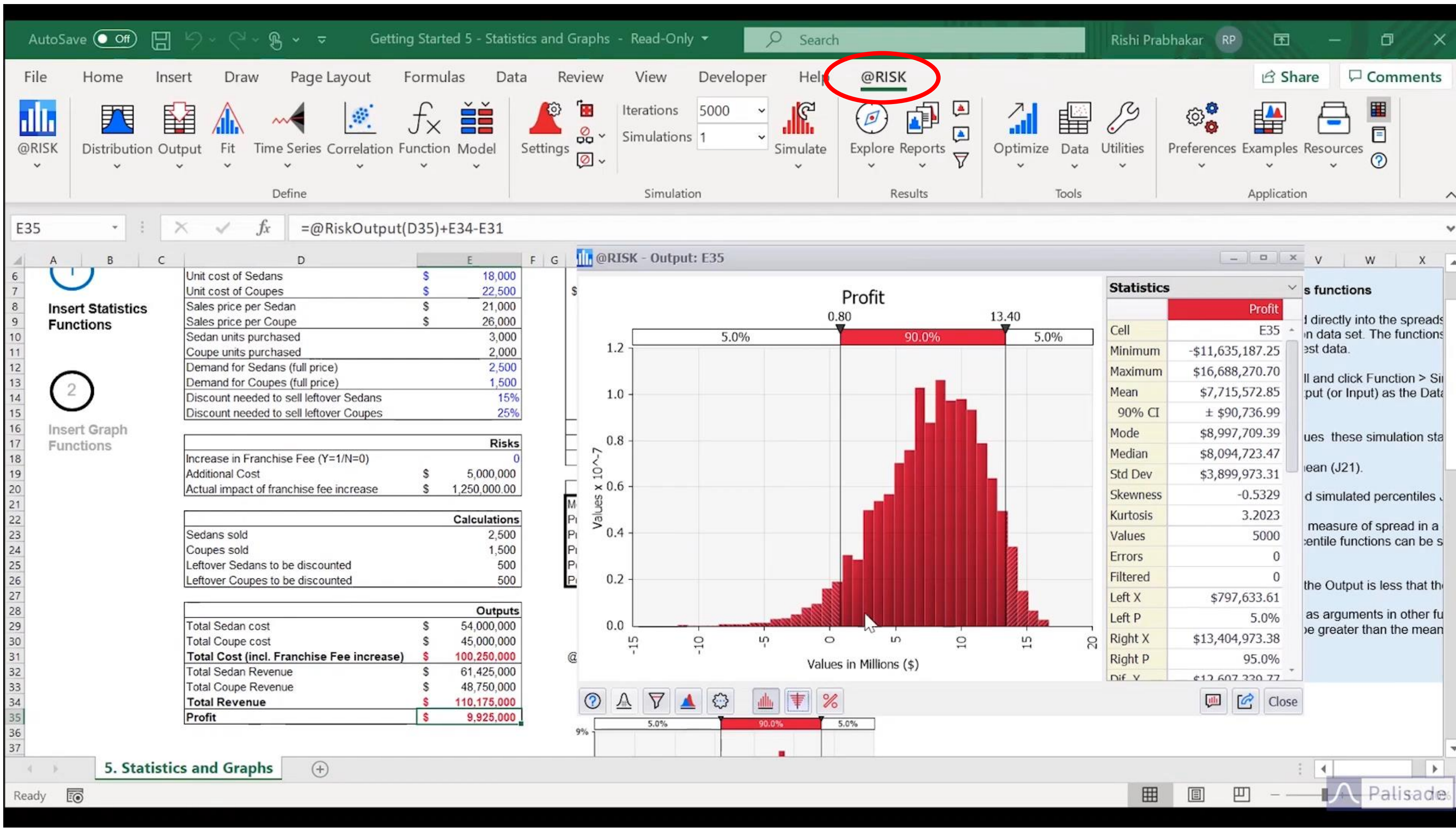
1395 Pounds/year

2096 Pounds/2 years

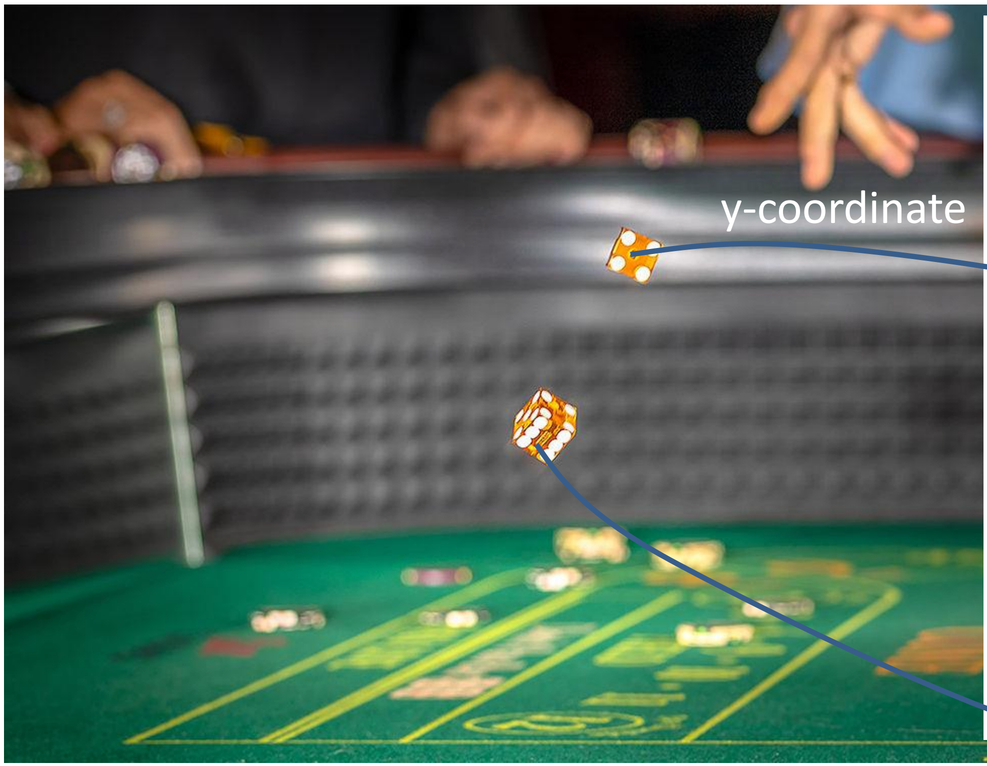
2694 Pounds/ 3 years

⇒ Not specialized for measurement uncertainty!

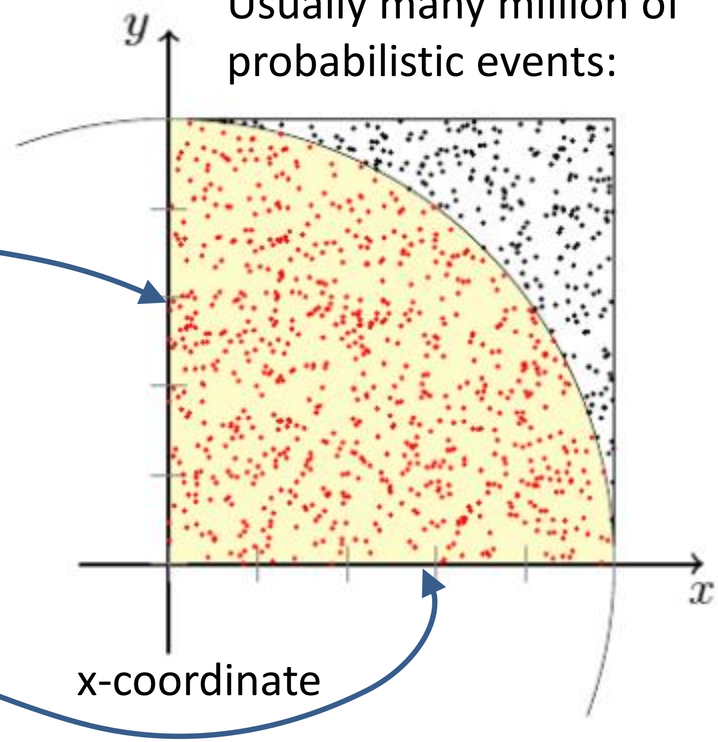
@Risk:


AutoSave Off
Getting Started 5 - Statistics and Graphs - Read-Only
Search
Rishi Prabhakar
File Home Insert Draw Page Layout Formulas Data Review View Developer Help @RISK
Share Comments
@RISK Distribution Output Fit Time Series Correlation Function Model
Define
Settings Iterations 5000 Simulations 1 Simulate
Simulation
Explore Reports
Results
Optimize Data Utilities
Tools
Preferences Examples Resources
Application
E35
=@RiskOutput(D35)+E34-E31
Insert Statistics Functions
Insert Graph Functions
Unit cost of Sedans $ 18,000
Unit cost of Coupes $ 22,500
Sales price per Sedan $ 21,000
Sales price per Coupe $ 26,000
Sedan units purchased 3,000
Coupe units purchased 2,000
Demand for Sedans (full price) 2,500
Demand for Coupes (full price) 1,500
Discount needed to sell leftover Sedans 15%
Discount needed to sell leftover Coupes 25%
Risks
Increase in Franchise Fee (Y=1/N=0) 0
Additional Cost $ 5,000,000
Actual impact of franchise fee increase $ 1,250,000.00
Calculations
Sedans sold 2,500
Coupes sold 1,500
Leftover Sedans to be discounted 500
Leftover Coupes to be discounted 500
Outputs
Total Sedan cost $ 54,000,000
Total Coupe cost $ 45,000,000
Total Cost (incl. Franchise Fee increase) $ 100,250,000
Total Sedan Revenue $ 61,425,000
Total Coupe Revenue $ 48,750,000
Total Revenue $ 110,175,000
Profit $ 9,925,000
@RISK - Output: E35
Profit
0.80 13.40
5.0% 90.0% 5.0%
Values x 10^-7
Values in Millions ($)
Statistics
Profit
Cell E35
Minimum -$11,635,187.25
Maximum $16,688,270.70
Mean $7,715,572.85
90% CI ± $90,736.99
Mode $8,997,709.39
Median $8,094,723.47
Std Dev $3,899,973.31
Skewness -0.5329
Kurtosis 3.2023
Values 5000
Errors 0
Filtered 0
Left X $797,633.61
Left P 5.0%
Right X $13,404,973.38
Right P 95.0%
Close
5. Statistics and Graphs
Ready
Palisade

Example for a Monte Carlo Simulation:



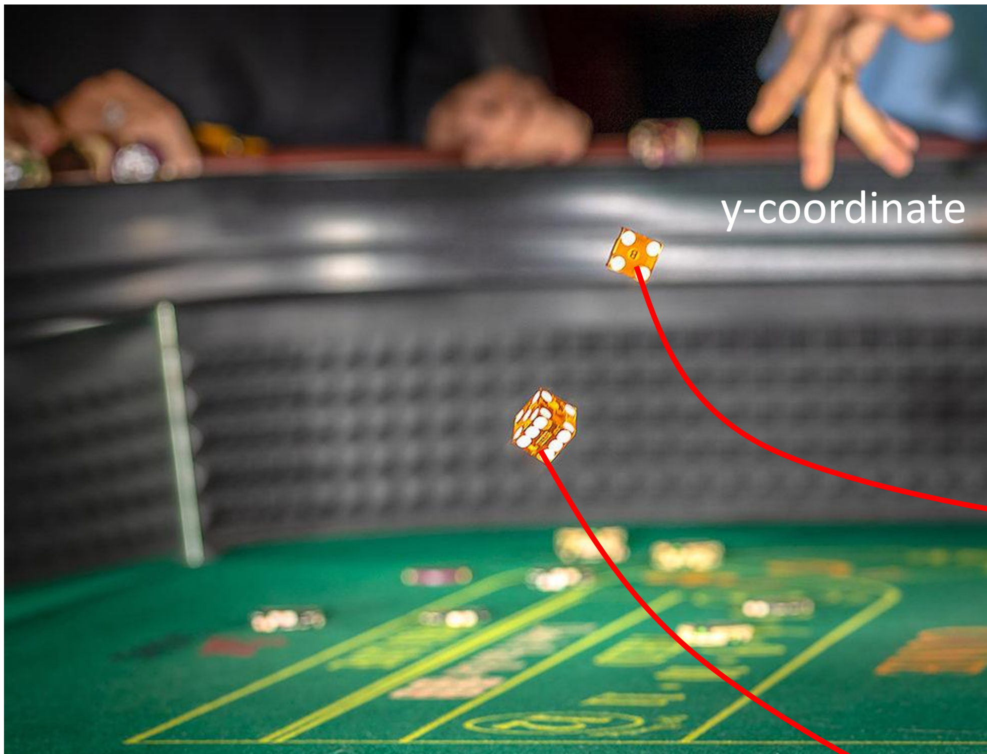
Usually many million of probabilistic events:



Equally distributed numbers!

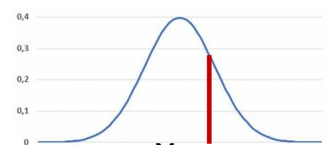
Yellow area = No. points inside / No. all points

Monte Carlo Simulation for Measurement Uncertainty:

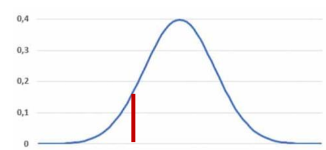


$y = f(x_1, x_2) = x_1 \cdot x_2$

$U(y) = ?$

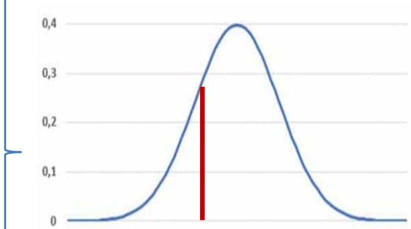


x_1



x_2

Result:



$x_1 \cdot x_2$

Normal distributed numbers!

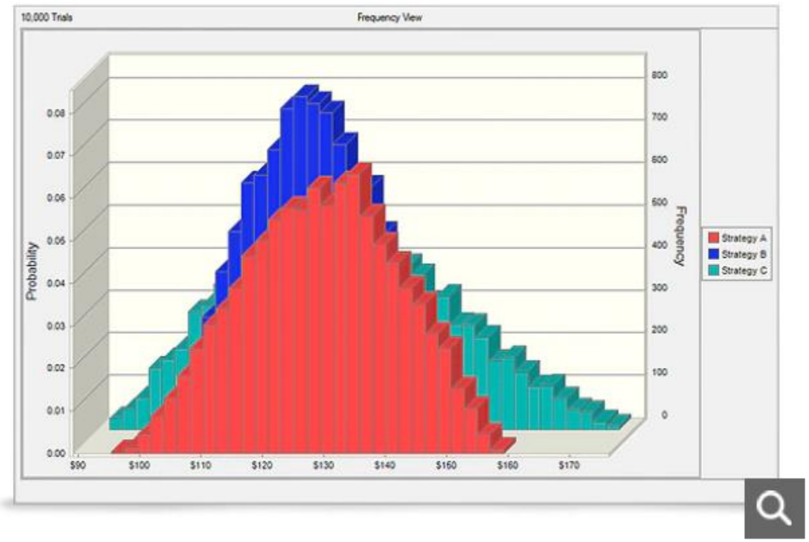
Crystal Ball:

Oracle Crystal Ball Product

Unparalleled Insight into the Critical Factors Affecting Risk

Oracle Crystal Ball is for strategic planners, financial analysts, engineers, scientists, entrepreneurs, CPAs, marketing managers, venture capitalists, consultants, Six Sigma professionals, and anyone else who uses spreadsheets to forecast uncertain results.

Request a demo



Understand variation in key performance indicators and risk-reward tradeoffs.

Price: 995 USD

Not specialized for measurement uncertainty!
Using Monte Carlo Simulation.

GUMsim:

GUMsim – Software for determination of measurement uncertainty

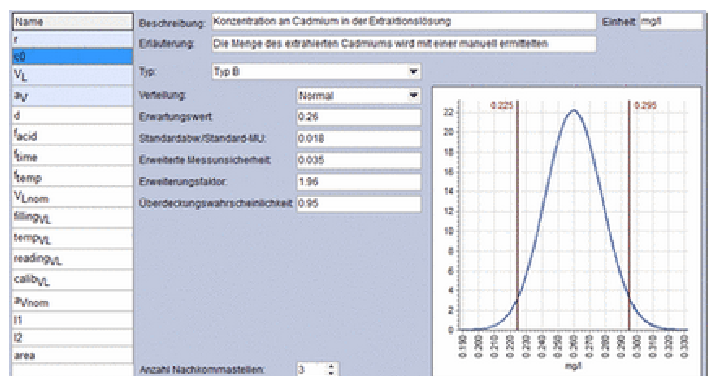
An ideal tool to help implement measurement uncertainty acc. to GUM and GUM S1

Deviations from the true value (measurement uncertainty) always accompany measurements carried out in the context of the evaluation or calibration of measurement instruments or procedures. Quality control requires that this uncertainty is quantified.

Based on the current Guide to the Expression of Uncertainty in Measurement (GUM) and the GUM supplement 1, **GUMsim®** is built upon advanced computational algorithms that allow more efficient determination of measurement uncertainty in compliance with **ISO/IEC 17025**.

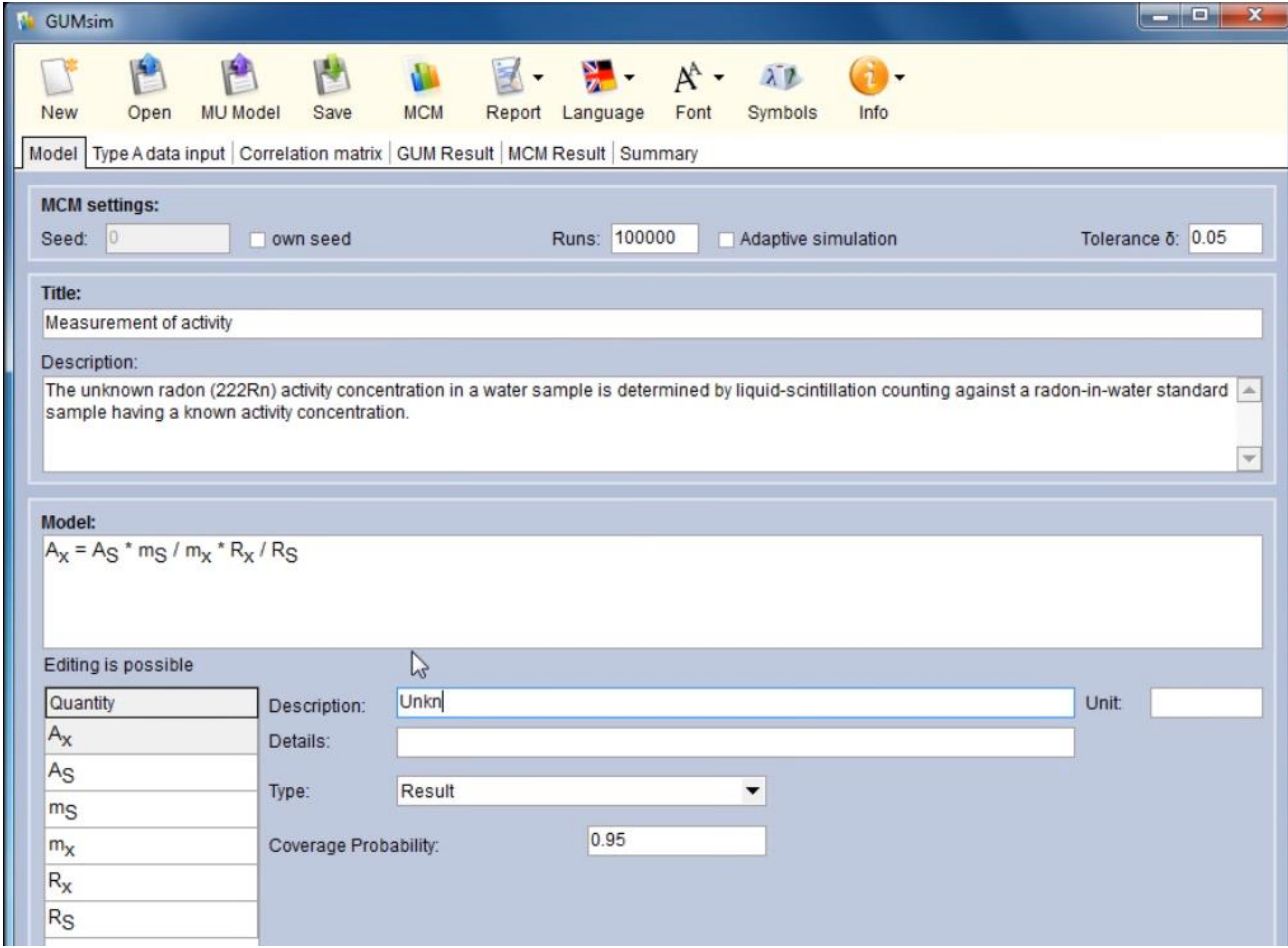
GUMsim's advantages at a glance:

- Fully developed statistical tool with excellent price/perfomance ratio
- Utilize **the Monte-Carlo method** for evaluation of measurement uncertainty
- Easy to use, intuitive interface
- Fast computation time
- Comprehensive and expert technical & statistical support service



Price: 840 €

Free trial version


GUMsim
New
Open
MU Model
Save
MCM
Report
Language
Font
Symbols
Info
Model
Type A data input
Correlation matrix
GUM Result
MCM Result
Summary
MCM settings:
Seed:
0
own seed
Runs:
100000
Adaptive simulation
Tolerance δ:
0.05
Title:
Measurement of activity
Description:
The unknown radon (222Rn) activity concentration in a water sample is determined by liquid-scintillation counting against a radon-in-water standard sample having a known activity concentration.
Model:
$A_X = A_S * m_S / m_X * R_X / R_S$
Editing is possible
Quantity
A_X
A_S
m_S
m_X
R_X
R_S
Description:
Unkn
Unit:
Details:
Type:
Result
Coverage Probability:
0.95

GUMsim:

Results according to GUM (JCGM 100:2008)

Result A_x

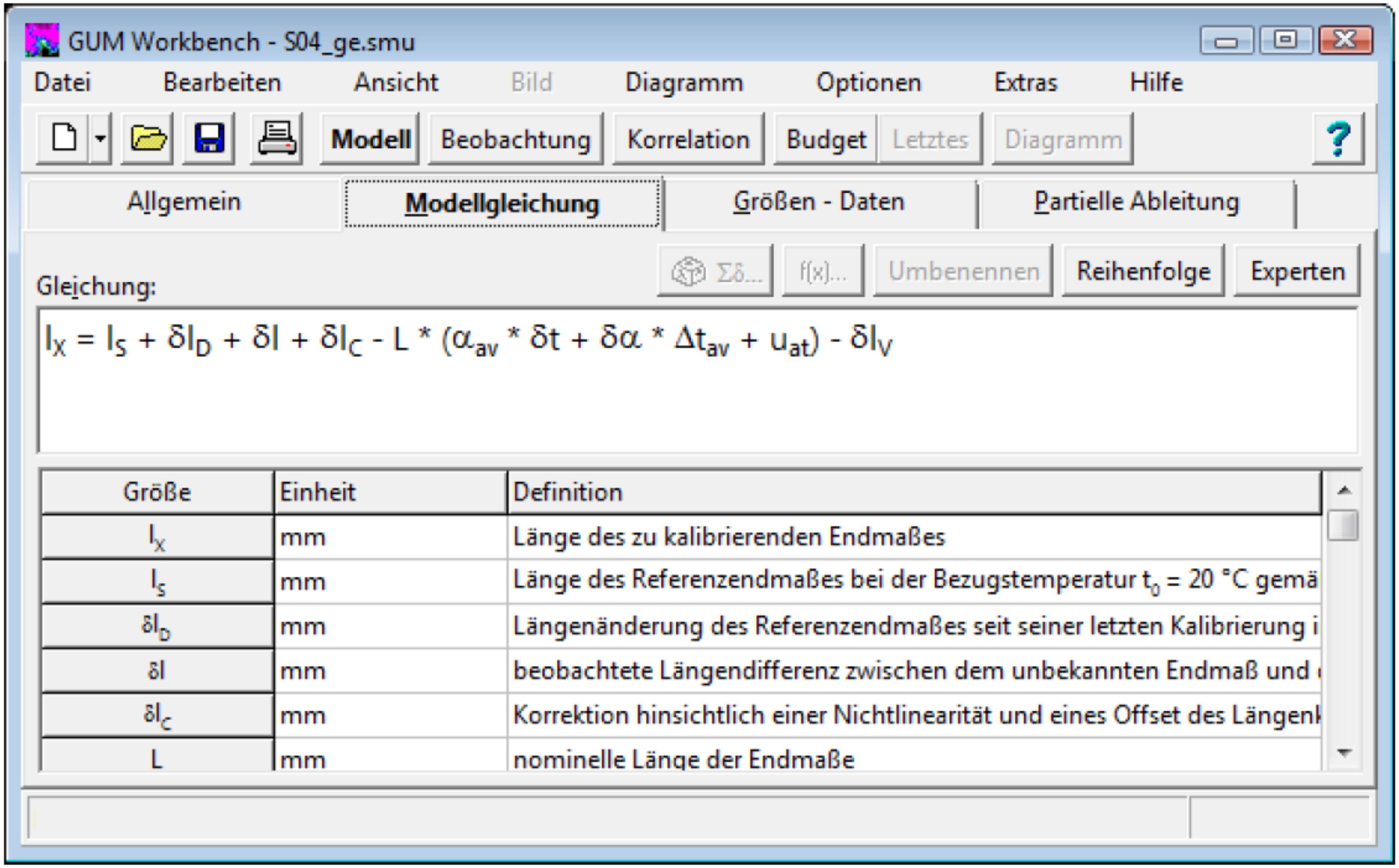
Value	Comb.std-unc.	Expanded uncertainty	Cov. factor	Cov. prob.
0.4299 Bq/g	0.0083 Bq/g	0.0163 Bq/g	1.9600	0.95

Uncertainty budget

Parameter	Value	Comb.std-unc.	Sensitivity	Uncertainty contribution	Index [%]
As	0.1368 Bq/g	0.0018 Bq/g	3.1429	0.0057 Bq/g	4605
ms	5.0192 g	0.0050 g	0.0857	0.0004 g	26
m_x	5.0571 g	0.0010 g	-0.0850	-85.0181e-6 g	1
R_x	652.6000	6.4157	0.0007	0.0042	-539
Rs	206.0883	3.7930	-0.0021	-0.0079	5906

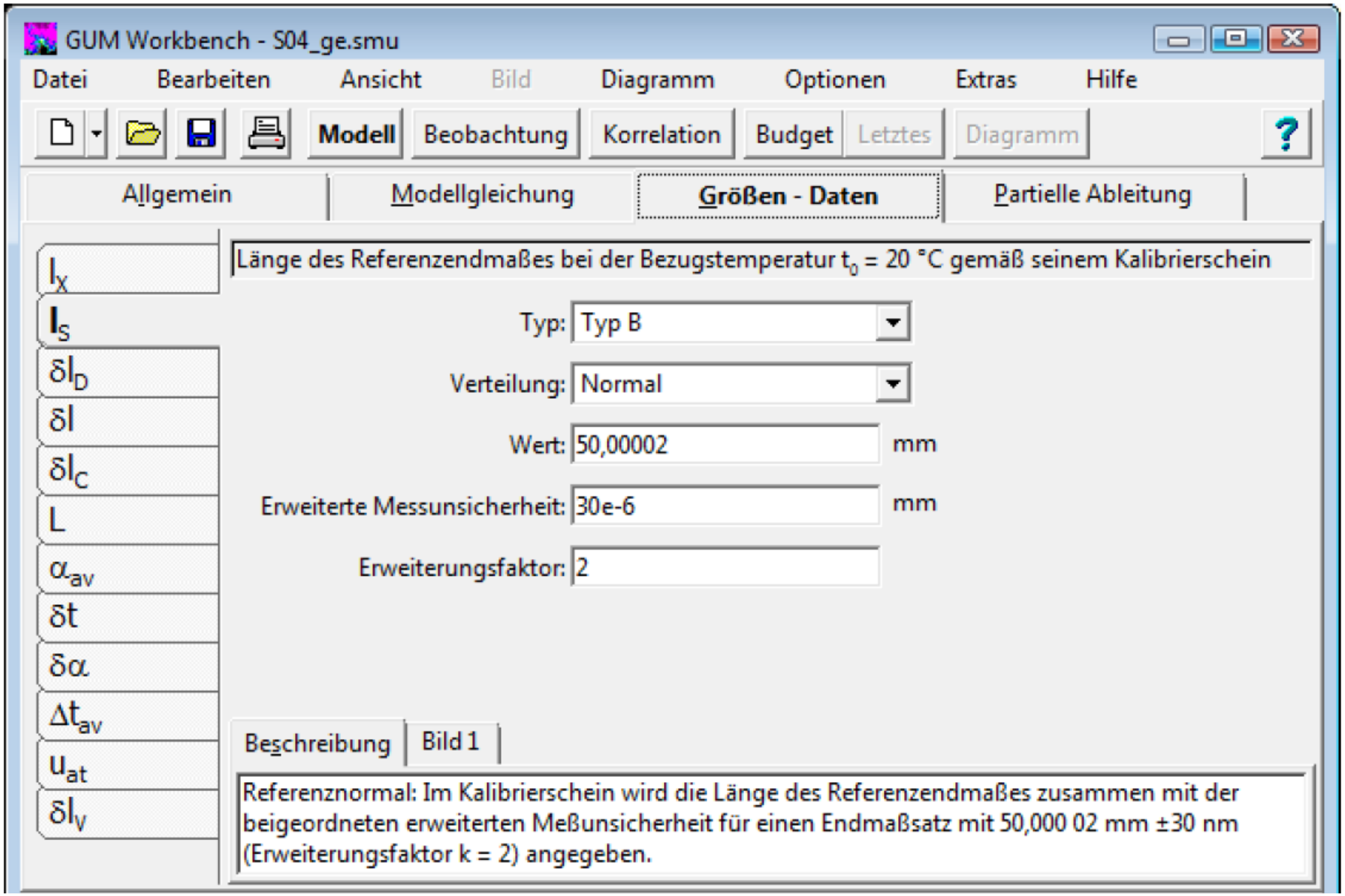
➡ https://www.youtube.com/watch?v=sbuuW1zqto8&list=PL50652C362E298AC7

GUM workbench:

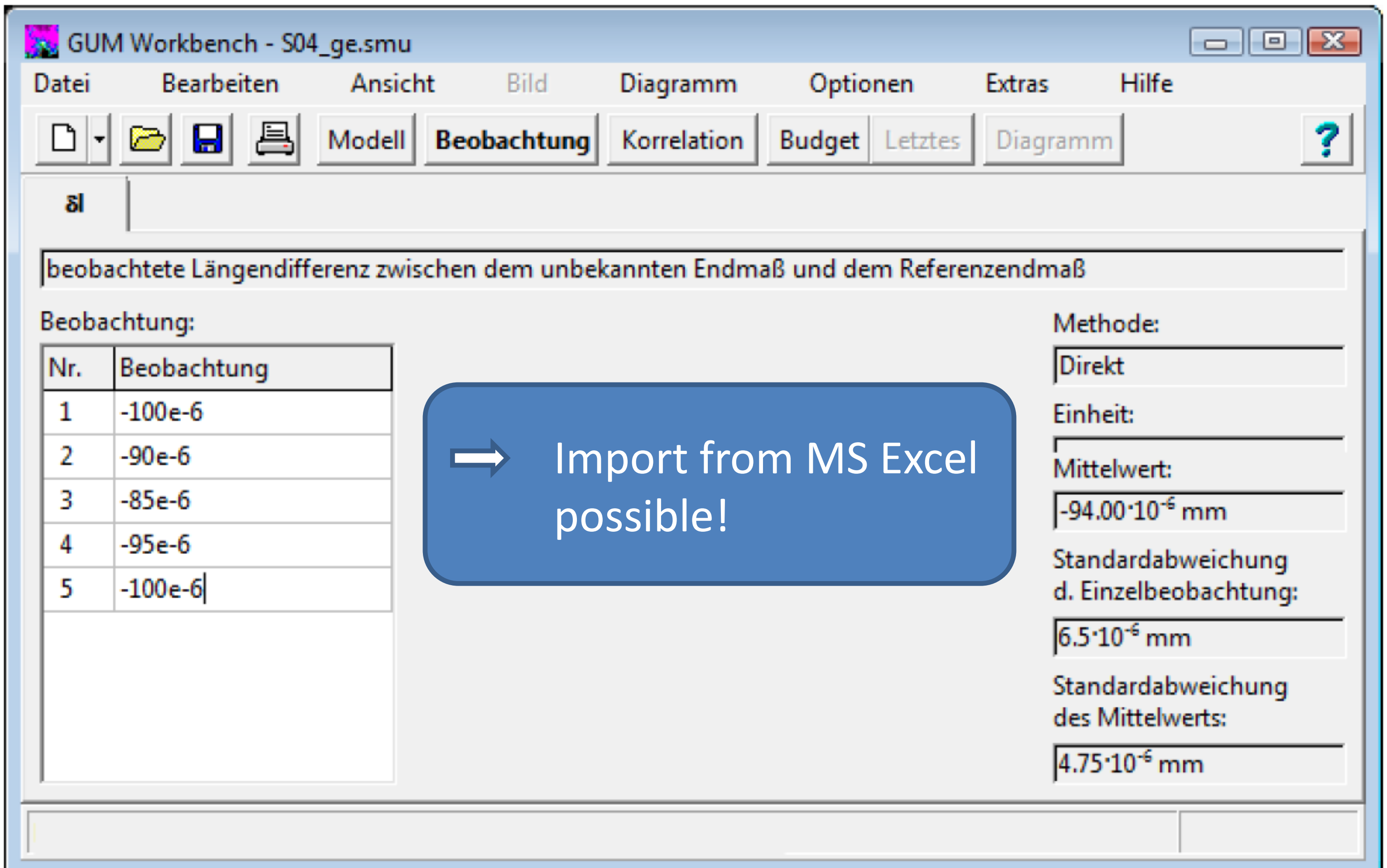


https://www.youtube.com/watch?v=emU5cx-qkTk

GUM workbench:

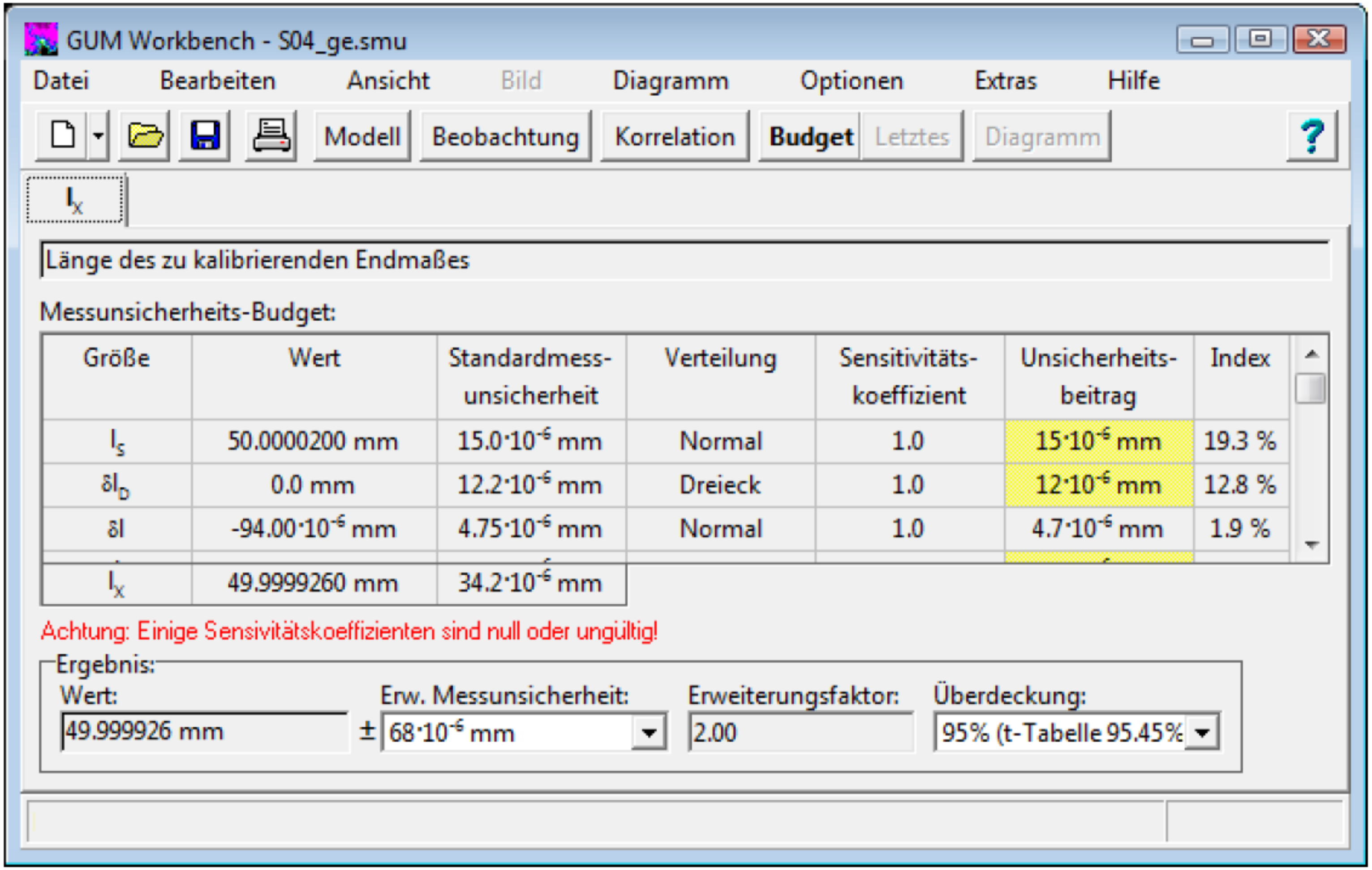


GUM workbench:



⇨ Import from MS Excel possible!

GUM workbench:



GUM workbench:

Metrodata GmbH | Datenverarbeitung für Meßtechnik und Qualitätssicherung

Price list

GUM Workbench can be ordered from Firma Metrodata GmbH.

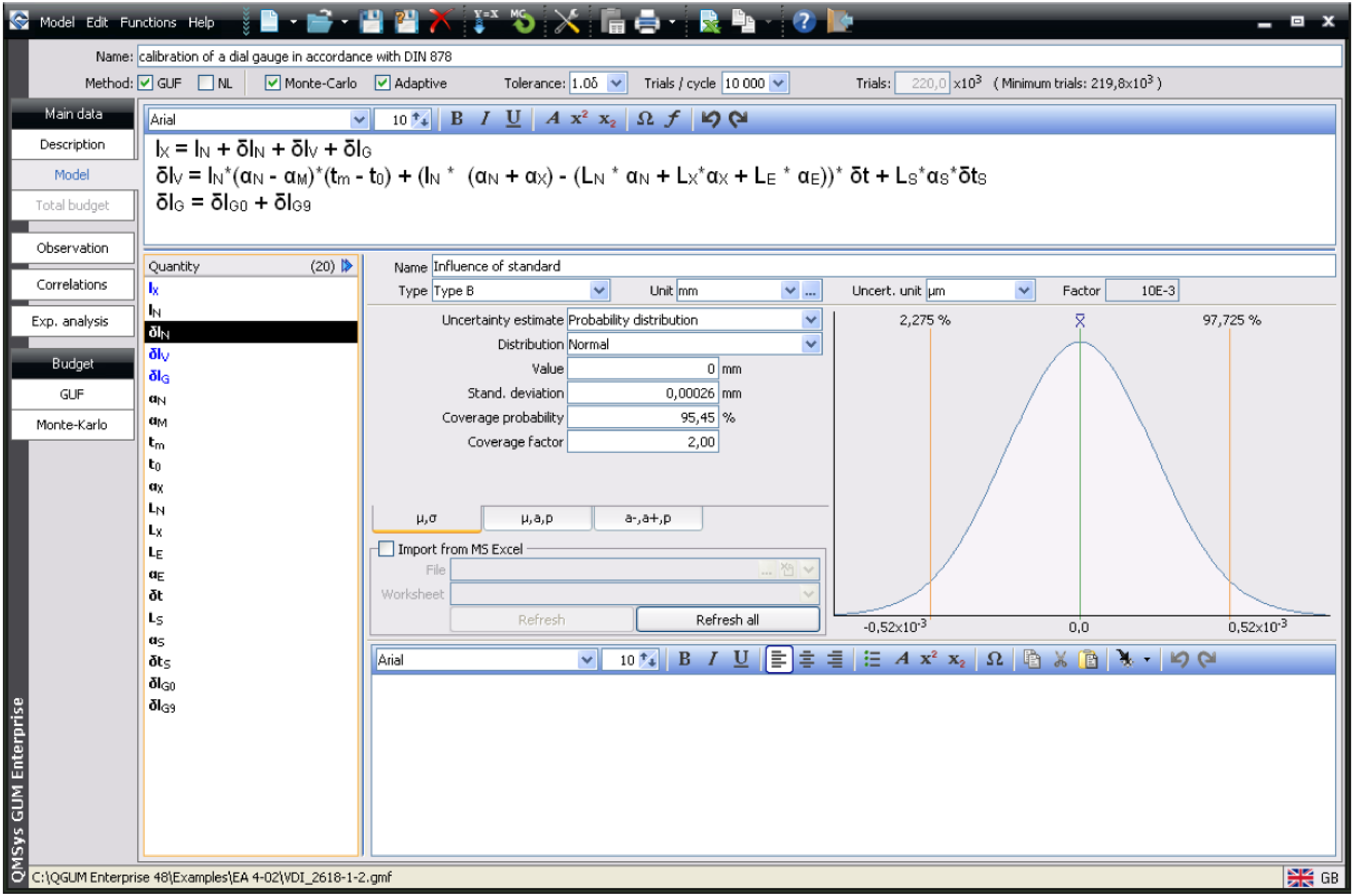
The price list is valid from March 1st 2019	excl. VAT	incl. 19% VAT
Full Version, 1 User		
GUM Workbench Standard Version 1.4	1700.00 EUR	2023.00 EUR
GUM Workbench Professional Version 2.4	2700.00 EUR	3213.00 EUR
Network licenses		
Version 1.4 for 5 separate users	6500.00 EUR	7735.00 EUR
Version 2.4 for 5 separate users	10600.00 EUR	12614.00 EUR
Version 2.4 with 2 shared licenses	5300.00 EUR	6307.00 EUR
Version 2.4 additional shared license	2200.00 EUR	2618.00 EUR
Prices for network licenses for more than 5 users on request		

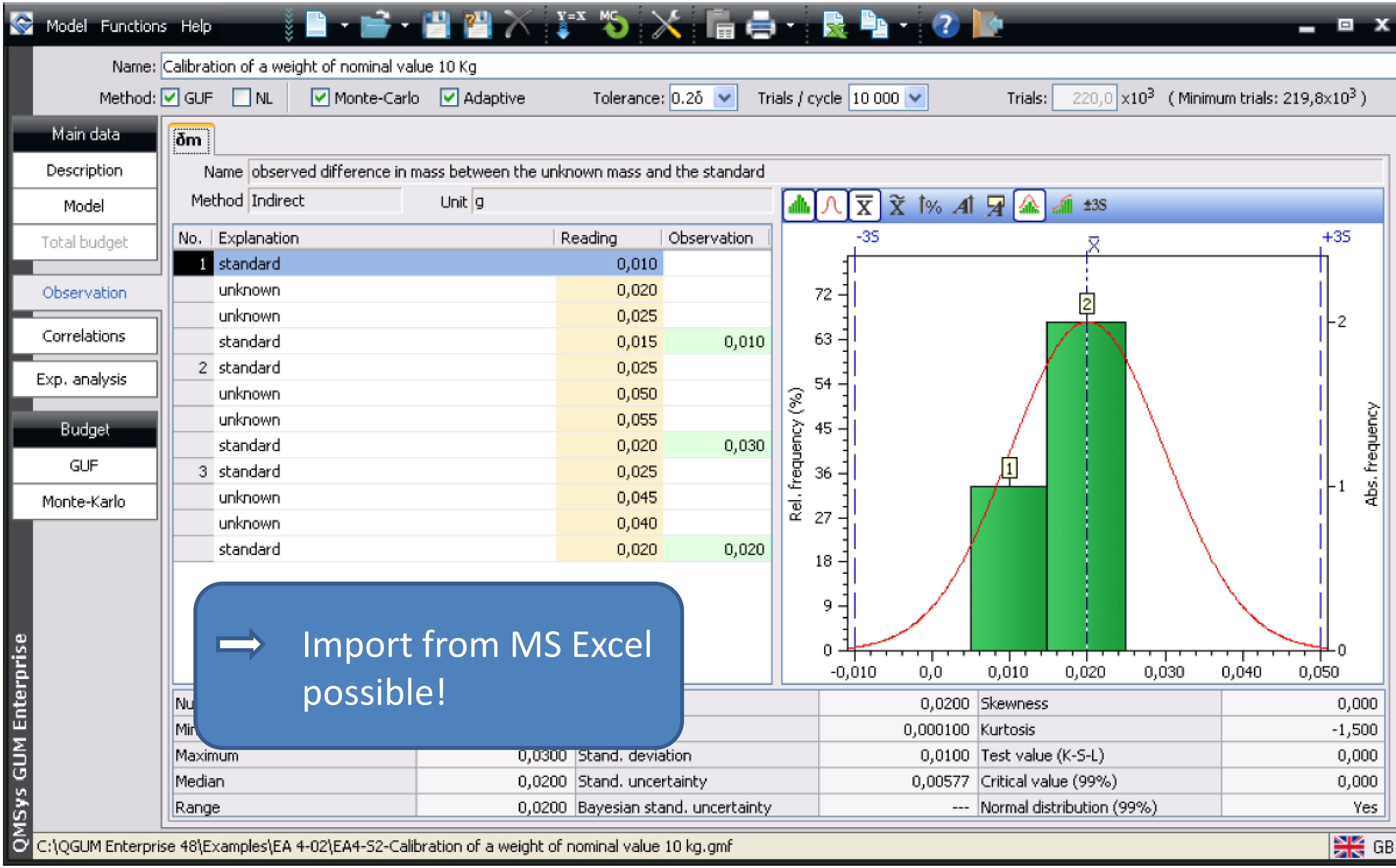
Available in:
- English
- French
- Spanish
- German

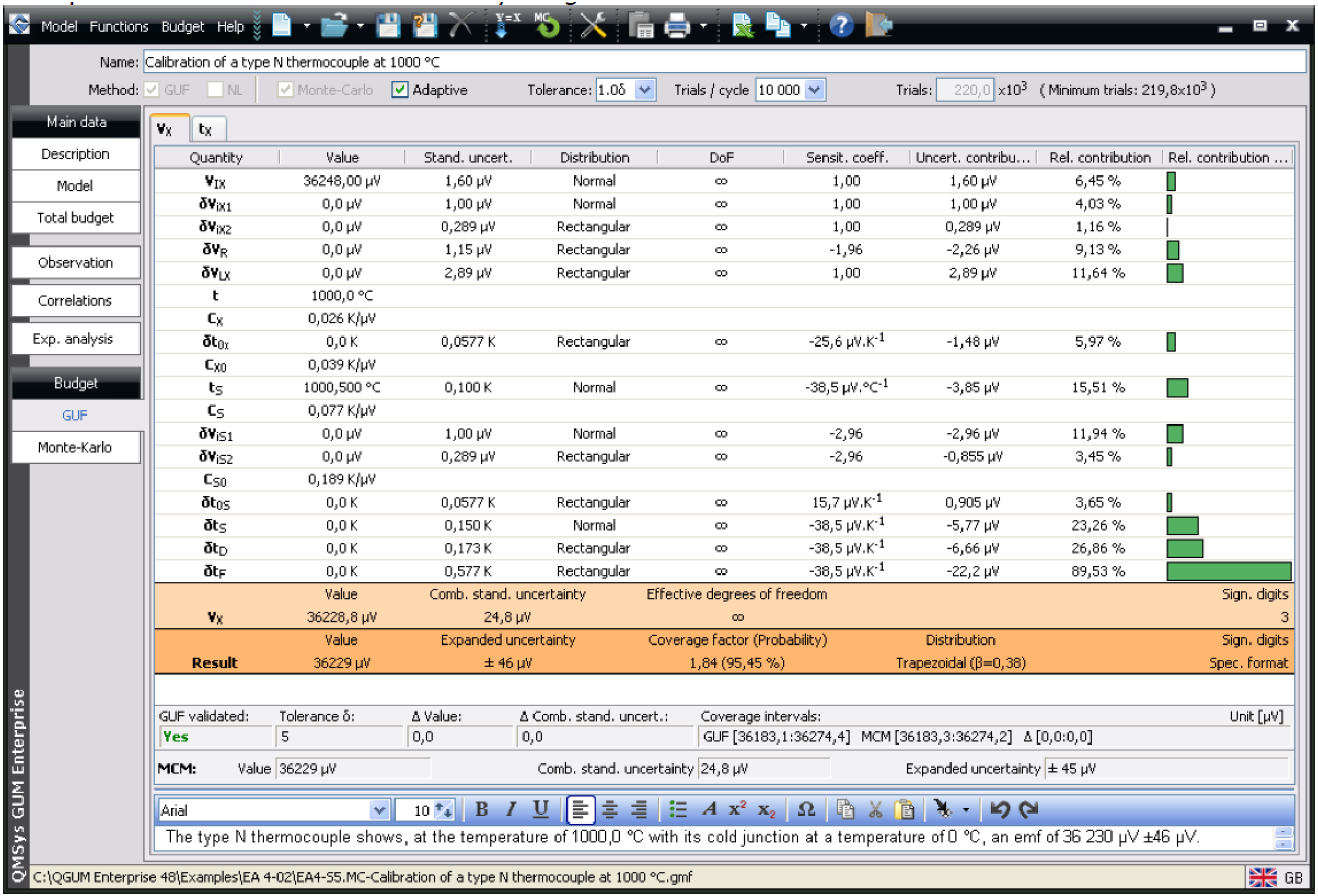
QMSys:

- ***GUM Enterprise*** is the ultimate software tool for precise and accurate analysis of the measurement uncertainty for all types of measurements. It uses three methods to calculate the measurement uncertainty - GUM method for linear models (uncertainty propagation), GUM method for nonlinear models (nonlinear sensitivity analysis, calculation of sensitivity indices of higher order) and Monte-Carlo method.
- ***GUM Professional*** is a cost-effective professional software that offers fast and reliable analysis of measurement uncertainty for linear and nonlinear models of the measurement process according to GUM Uncertainty Framework and validation of the results using the Monte Carlo method.
- ***GUM Standard*** offers the classical calculation of the measurement uncertainty for linear and quasi-linear models of the measurement process according to GUM Uncertainty Framework.
- ***GUM Educational*** has a limited functionality (1 output quantity, maximum 10 input quantities and maximum 50 measurements for Type A) and can be freely used for educational purposes, seminars and workshops.

Single-User Versions and Portable Versions on USB Memory Stick				
Number of Licenses	**1-2**	**3-5**	**6-10**	**> 10**
GUM Enterprise	1200.00 €	1140.00 €	1080.00 €	1020.00 €
GUM Professional	1000.00 €	950.00 €	900.00 €	850.00 €
GUM Calculator ⟹ Standard?	400.00 €	380.00 €	360.00 €	340.00 €
GUM Excel Add-in	300.00 €	285.00 €	270.00 €	255.00 €


Model Edit Functions Help
Name: calibration of a dial gauge in accordance with DIN 878
Method: GUF NL Monte-Carlo Adaptive
Tolerance: 1.0δ
Trials / cycle 10 000
Trials: 220,0 x10³ (Minimum trials: 219,8x10³)
Main data
Description
Model
Total budget
Observation
Correlations
Exp. analysis
Budget
GUF
Monte-Karlo
lX = lN + δlN + δlV + δlG
δlV = lN*(αN - αM)*(tm - t0) + (lN * (αN + αX) - (LN * αN + LX*αX + LE * αE))* δt + LS*αS*δtS
δlG = δlG0 + δlG9
Quantity (20)
lX
lN
δlN
δlV
δlG
αN
αM
tm
t0
αX
LN
LX
LE
αE
δt
LS
αS
δtS
δlG0
δlG9
Name Influence of standard
Type Type B
Unit mm
Uncert. unit µm
Factor 10E-3
Uncertainty estimate Probability distribution
Distribution Normal
Value 0 mm
Stand. deviation 0,00026 mm
Coverage probability 95,45 %
Coverage factor 2,00
µ,σ
µ,a,p
a-,a+,p
Import from MS Excel
File
Worksheet
Refresh
Refresh all
2,275 %
97,725 %
-0,52x10⁻³
0,0
0,52x10⁻³
QMSys GUM Enterprise
C:\QGUM Enterprise 48\Examples\EA 4-02\VDI_2618-1-2.gmf
GB


Import from MS Excel possible!
Model Functions Help
Name: Calibration of a weight of nominal value 10 Kg
Method: GUF NL Monte-Carlo Adaptive Tolerance: 0.2δ Trials / cycle 10 000 Trials: 220,0 x10³ (Minimum trials: 219,8x10³)
Main data
Description
Model
Total budget
Observation
Correlations
Exp. analysis
Budget
GUF
Monte-Karlo
δm
Name observed difference in mass between the unknown mass and the standard
Method Indirect Unit g
No. Explanation Reading Observation
1 standard 0,010
unknown 0,020
unknown 0,025
standard 0,015 0,010
2 standard 0,025
unknown 0,050
unknown 0,055
standard 0,020 0,030
3 standard 0,025
unknown 0,045
unknown 0,040
standard 0,020 0,020
Rel. frequency (%)
Abs. frequency
-3S +3S
Maximum 0,0300 Stand. deviation 0,0100 Test value (K-S-L) 0,000
Median 0,0200 Stand. uncertainty 0,00577 Critical value (99%) 0,000
Range 0,0200 Bayesian stand. uncertainty --- Normal distribution (99%) Yes
0,0200 Skewness 0,000
0,000100 Kurtosis -1,500
QMSys GUM Enterprise
C:\QGUM Enterprise 48\Examples\EA 4-02\EA4-S2-Calibration of a weight of nominal value 10 kg.gmf
GB



Model Functions Budget Help

Name: Calibration of a type N thermocouple at 1000 °C

Method: GUF NL Monte-Carlo Adaptive Tolerance: 1.0δ Trials / cycle 10 000 Trials: 220,0 x10³ (Minimum trials: 219,8x10³)

Main data: Description, Model, Total budget, Observation, Correlations, Exp. analysis

Budget: GUF, Monte-Karlo

Tabs: V_X, t_X

Quantity	Value	Stand. uncert.	Distribution	DoF	Sensit. coeff.	Uncert. contribu...	Rel. contribution	Rel. contribution ...
V_{iX}	36248,00 µV	1,60 µV	Normal	∞	1,00	1,60 µV	6,45 %	
δV_{iX1}	0,0 µV	1,00 µV	Normal	∞	1,00	1,00 µV	4,03 %	
δV_{iX2}	0,0 µV	0,289 µV	Rectangular	∞	1,00	0,289 µV	1,16 %	
δV_R	0,0 µV	1,15 µV	Rectangular	∞	-1,96	-2,26 µV	9,13 %	
δV_{LX}	0,0 µV	2,89 µV	Rectangular	∞	1,00	2,89 µV	11,64 %	
t	1000,0 °C							
C_X	0,026 K/µV							
δt_{0X}	0,0 K	0,0577 K	Rectangular	∞	-25,6 µV.K⁻¹	-1,48 µV	5,97 %	
C_{X0}	0,039 K/µV							
t_S	1000,500 °C	0,100 K	Normal	∞	-38,5 µV.°C⁻¹	-3,85 µV	15,51 %	
C_S	0,077 K/µV							
δV_{iS1}	0,0 µV	1,00 µV	Normal	∞	-2,96	-2,96 µV	11,94 %	
δV_{iS2}	0,0 µV	0,289 µV	Rectangular	∞	-2,96	-0,855 µV	3,45 %	
C_{S0}	0,189 K/µV							
δt_{0S}	0,0 K	0,0577 K	Rectangular	∞	15,7 µV.K⁻¹	0,905 µV	3,65 %	
δt_S	0,0 K	0,150 K	Normal	∞	-38,5 µV.K⁻¹	-5,77 µV	23,26 %	
δt_D	0,0 K	0,173 K	Rectangular	∞	-38,5 µV.K⁻¹	-6,66 µV	26,86 %	
δt_F	0,0 K	0,577 K	Rectangular	∞	-38,5 µV.K⁻¹	-22,2 µV	89,53 %	

	Value	Comb. stand. uncertainty	Effective degrees of freedom		Sign. digits
V_X	36228,8 µV	24,8 µV	∞		3

	Value	Expanded uncertainty	Coverage factor (Probability)	Distribution	Sign. digits
Result	36229 µV	± 46 µV	1,84 (95,45 %)	Trapezoidal (β=0,38)	Spec. format

GUF validated:	Tolerance δ:	Δ Value:	Δ Comb. stand. uncert.:	Coverage intervals:	Unit [µV]
Yes	5	0,0	0,0	GUF [36183,1:36274,4] MCM [36183,3:36274,2] Δ [0,0:0,0]	

MCM: Value 36229 µV Comb. stand. uncertainty 24,8 µV Expanded uncertainty ± 45 µV

The type N thermocouple shows, at the temperature of 1000,0 °C with its cold junction at a temperature of 0 °C, an emf of 36 230 µV ±46 µV.

C:\QGUM Enterprise 48\Examples\EA 4-02\EA4-S5.MC-Calibration of a type N thermocouple at 1000 °C.gmf

QMSys GUM Enterprise

Uncertainty Sidekick Pro:

Uncertainty Sidekick Pro Purchasing Options

Single-User License

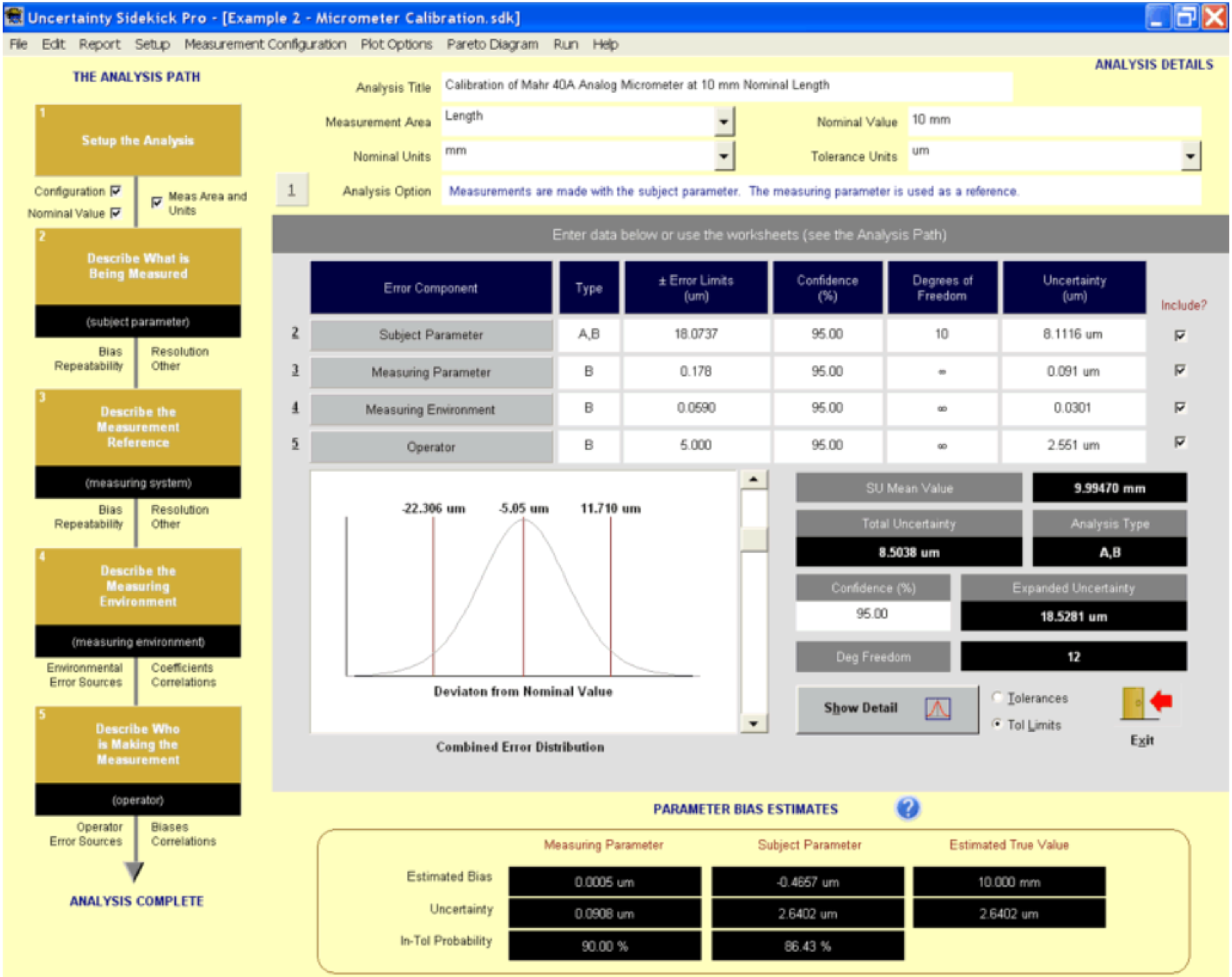
The **$295** single-user software license includes one (1) PC setup CD and one (1) document set.

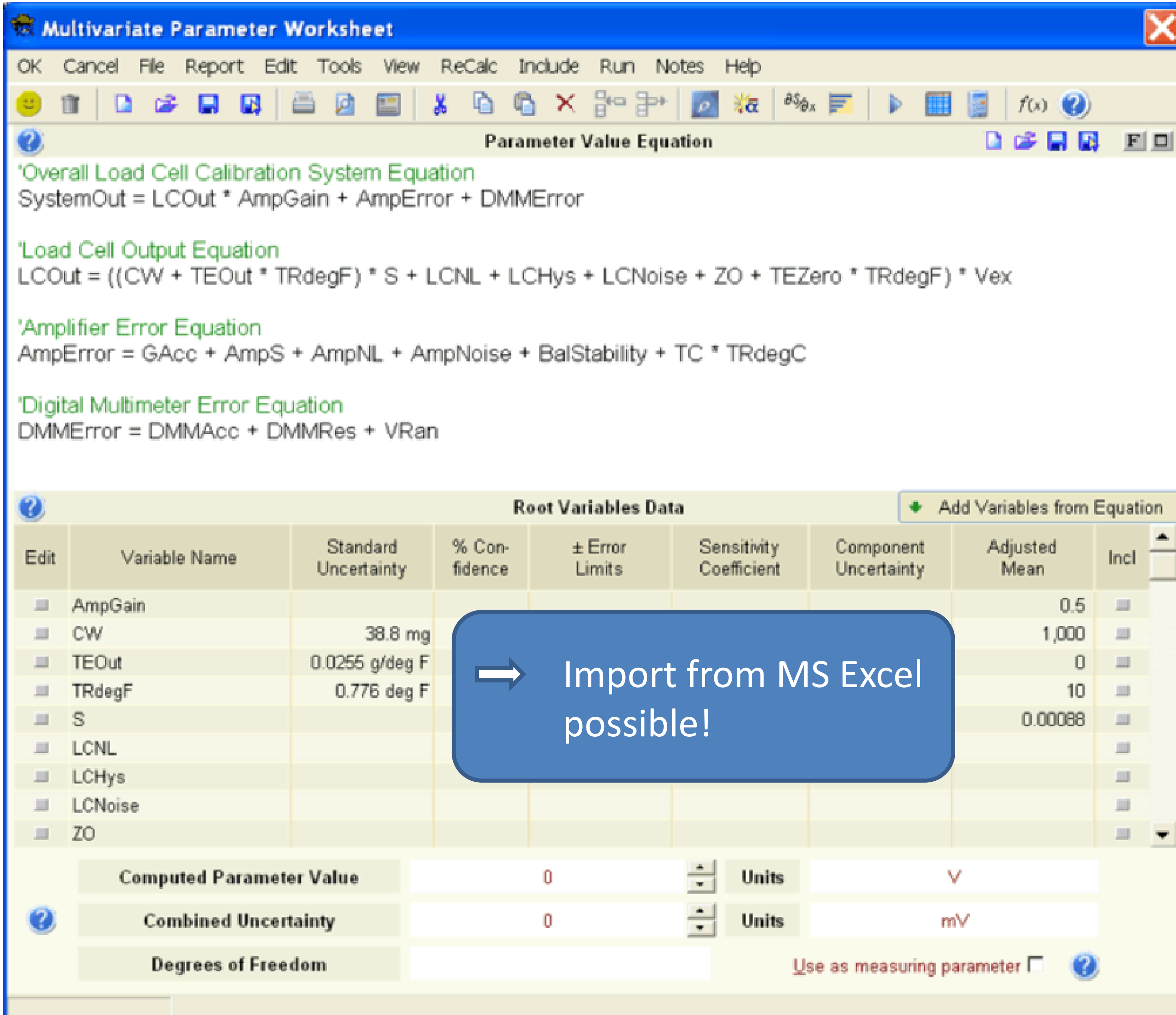
Network License

The Uncertainty Sidekick Pro network license is available for six (6) or more registered users. The network license option includes one (1) network server setup CD, six (6) or more PC client setup CD's, and six (6) or more user document sets.

Single-Computer, Multi-User License

This **$795** software license for up to 5 registered users includes one (1) setup CD for installation on a shared workstation computer and one (1) documentation set.


Uncertainty Sidekick Pro - [Example 2 - Micrometer Calibration.sdk]
File Edit Report Setup Measurement Configuration Plot Options Pareto Diagram Run Help
THE ANALYSIS PATH
1 Setup the Analysis
Configuration
Nominal Value
Meas Area and Units
2 Describe What is Being Measured
(subject parameter)
Bias Repeatability
Resolution Other
3 Describe the Measurement Reference
(measuring system)
Bias Repeatability
Resolution Other
4 Describe the Measuring Environment
(measuring environment)
Environmental Error Sources
Coefficients Correlations
5 Describe Who is Making the Measurement
(operator)
Operator Error Sources
Biases Correlations
ANALYSIS COMPLETE
ANALYSIS DETAILS
Analysis Title
Calibration of Mahr 40A Analog Micrometer at 10 mm Nominal Length
Measurement Area
Length
Nominal Value
10 mm
Nominal Units
mm
Tolerance Units
um
1
Analysis Option
Measurements are made with the subject parameter. The measuring parameter is used as a reference.
Enter data below or use the worksheets (see the Analysis Path)
Error Component
Type
± Error Limits (um)
Confidence (%)
Degrees of Freedom
Uncertainty (um)
Include?
2 Subject Parameter A,B 18.0737 95.00 10 8.1116 um
3 Measuring Parameter B 0.178 95.00 ∞ 0.091 um
4 Measuring Environment B 0.0590 95.00 ∞ 0.0301
5 Operator B 5.000 95.00 ∞ 2.551 um
-22.306 um
-5.05 um
11.710 um
Deviaton from Nominal Value
Combined Error Distribution
SU Mean Value
9.99470 mm
Total Uncertainty
8.5038 um
Analysis Type
A,B
Confidence (%)
95.00
Expanded Uncertainty
18.5281 um
Deg Freedom
12
Show Detail
Tolerances
Tol Limits
Exit
PARAMETER BIAS ESTIMATES
Measuring Parameter
Subject Parameter
Estimated True Value
Estimated Bias 0.0005 um -0.4657 um 10.000 mm
Uncertainty 0.0908 um 2.6402 um 2.6402 um
In-Tol Probability 90.00 % 86.43 %


Multivariate Parameter Worksheet
OK Cancel File Report Edit Tools View ReCalc Include Run Notes Help
Parameter Value Equation
'Overall Load Cell Calibration System Equation
SystemOut = LCOut * AmpGain + AmpError + DMMError
'Load Cell Output Equation
LCOut = ((CW + TEOut * TRdegF) * S + LCNL + LCHys + LCNoise + ZO + TEZero * TRdegF) * Vex
'Amplifier Error Equation
AmpError = GAcc + AmpS + AmpNL + AmpNoise + BalStability + TC * TRdegC
'Digital Multimeter Error Equation
DMMError = DMMAcc + DMMRes + VRan
Root Variables Data
Add Variables from Equation
Edit
Variable Name
Standard Uncertainty
% Confidence
± Error Limits
Sensitivity Coefficient
Component Uncertainty
Adjusted Mean
Incl
AmpGain
0.5
CW
38.8 mg
1,000
TEOut
0.0255 g/deg F
0
TRdegF
0.776 deg F
10
S
0.00088
LCNL
LCHys
LCNoise
ZO
Computed Parameter Value
0
Units
V
Combined Uncertainty
0
Units
mV
Degrees of Freedom
Use as measuring parameter
Import from MS Excel possible!

Uncertainty Sidekick Print Preview

Close Copy Run

1/1

Uncertainty Analysis Report **Measurement Uncertainty**

Calibration of ThermoProbe TL-1A Digital Thermometer at 100 deg C

ACME Industries - Bakersfield

08-Sep-2006

Submitted:
Bob King
Engineer, Test & Evaluation
6/1/05

File Name: Example 3 - Thermometer Calibration.sdk

Subject Unit

Manufacturer:	ThermoProbe, Inc.
Model Number:	TL-1A
Description:	-43 to 315 deg C Digital Thermometer
Measured Quantity:	Thermometer Temperature Reading
Estimated Parameter Value:	99.99600 deg C

Measuring Unit

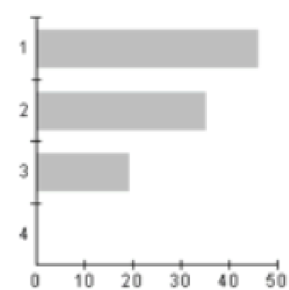
Manufacturer:	Hart Scientific
Model Number:	5699
Description:	High-Temperature Metal-Sheath SPRT
Measured Quantity:	SPRT Temperature Reading

Analysis Results

Uncertainty Component	Standard Uncertainty (deg C)	Confidence Level	Deg. Freedom	Confidence Limits (deg C)	Type
Measurand	0.0058	95.00	16	0.0123	A,B
Reference	0.00240	95.00	105	0.00476	A,B
Environment	0.0044	95.00	Infinite	0.0086	B
Operator Bias	0.0	95.00			
Combined Uncertainty	**0.00767 deg C**		**49**		**A,B**

Pareto Diagram

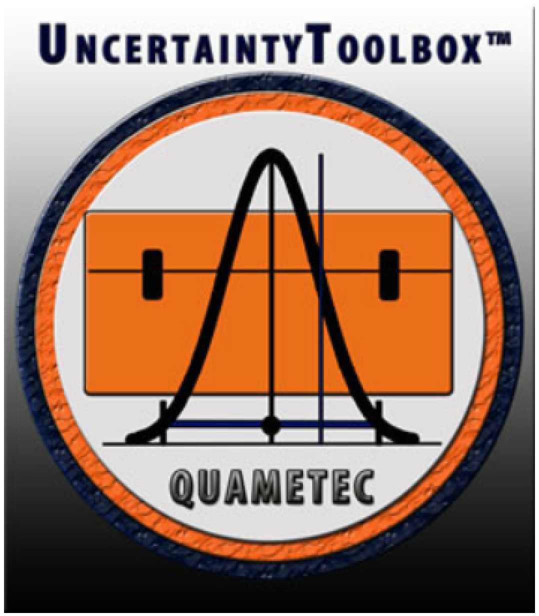
Rank	Error Component	Type	Weight (%)
1	Measurand	A,B	46.032
2	Environment	B	34.921
3	Reference	A,B	19.048



Uncertainty Toolbox:

QUAMETEC™

Home > Site pages > UncertaintyToolbox™ add-in for Microsoft® Excel®



The #1 Choice of Accredited Laboratories in America!

UncertaintyToolbox™ Addin for Microsoft® Excel® 64 & 32Bit 2010, 2013, 2016, 2019 & Office365- Desktop Version

Current Version 6.22 Released 28-Mar-2021

UncertaintyToolbox™ NOW includes Specific and Global Risk Analysis Tools:

1) ILAC G8/09:2019 and ISO14253-1:2017 compliant False Accept Risk and False Reject Risk Analysis and Management Templates

2) ANSI/NCSL Z540.3 complaint 2% FAR Management Template

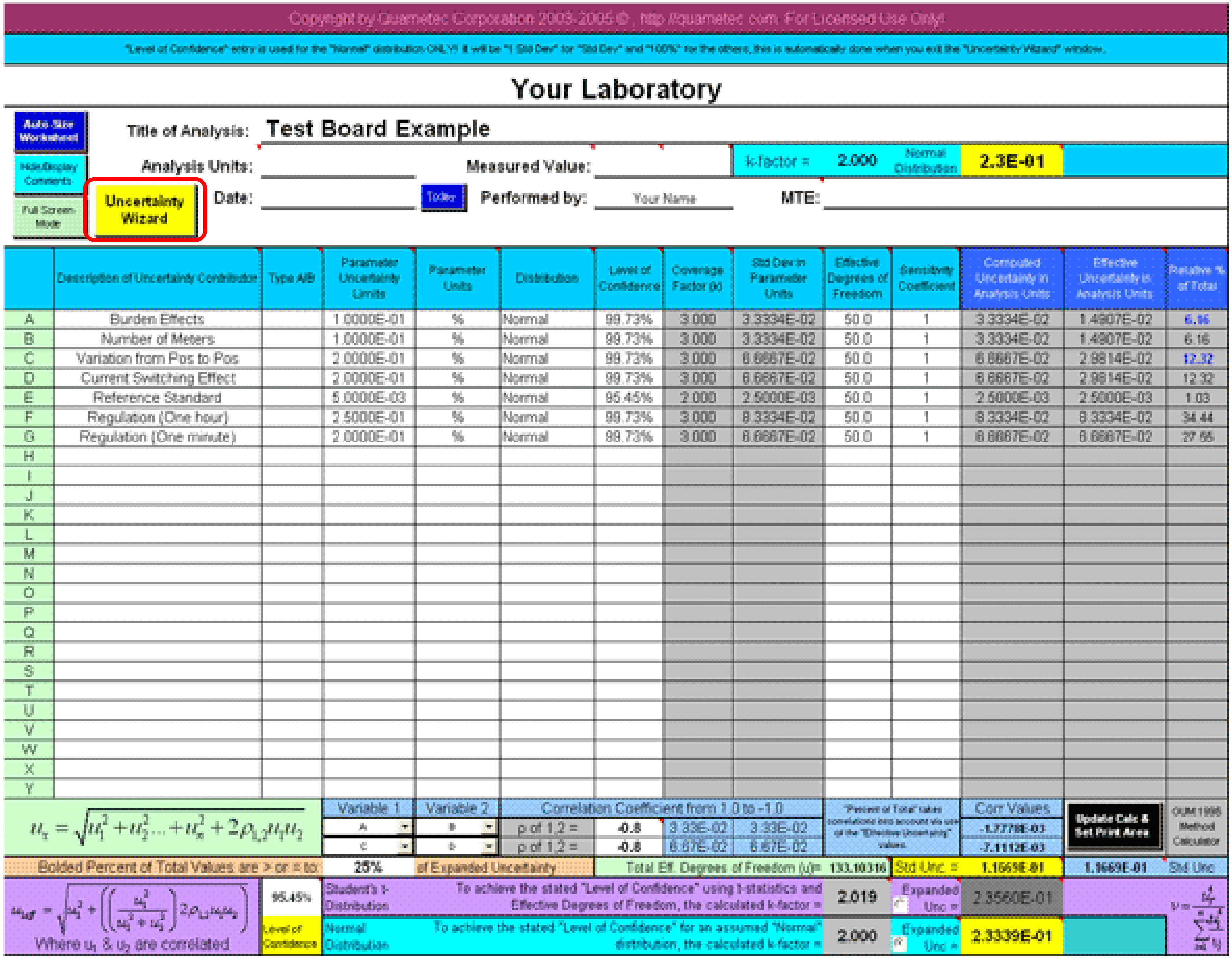
Risk Management Templates link directly with the applicable measurement uncertainty estimate ensuring the latest estimate of uncertainty is applied to the resulting decision risk analysis.

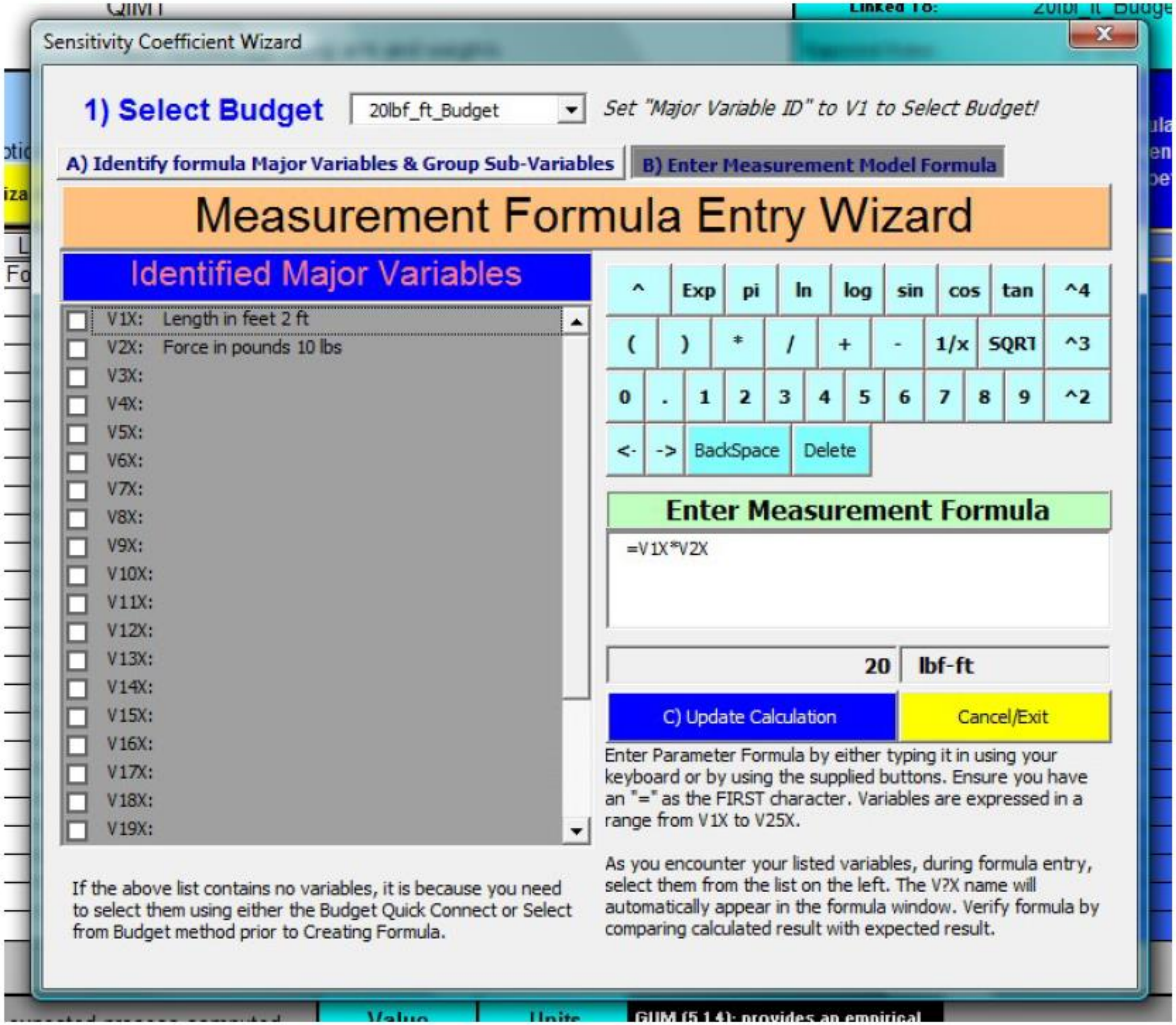
Uncertainty Toolbox:

Pricing for *UncertaintyToolbox™ User License*

US Dollars

Number of Users	Software Download w/ 1-Year Support**	Annual Software Support** with Free Upgrades	* Software Upgrade Fee w/1-year Software Support**
1 user license	$ 995 per user	$ 250 per user/year	$ 795 *per user
2 or more	$ 935 per user	$ 250 per user/year	$ 795 *per user


Copyright by Quametec Corporation 2003-2005 ©, http://quametec.com For Licensed Use Only!
Your Laboratory
Auto-Size Worksheet
Hide/Display Comments
Full Screen Mode
Uncertainty Wizard
Title of Analysis: Test Board Example
Analysis Units:
Measured Value:
k-factor = 2.000
Normal Distribution
2.3E-01
Date:
Today
Performed by: Your Name
MTE:
Description of Uncertainty Contributor
Type A/B
Parameter Uncertainty Limits
Parameter Units
Distribution
Level of Confidence
Coverage Factor (k)
Std Dev in Parameter Units
Effective Degrees of Freedom
Sensitivity Coefficient
Computed Uncertainty in Analysis Units
Effective Uncertainty in Analysis Units
Relative % of Total
A Burden Effects 1.0000E-01 % Normal 99.73% 3.000 3.3334E-02 50.0 1 3.3334E-02 1.4907E-02 6.16
B Number of Meters 1.0000E-01 % Normal 99.73% 3.000 3.3334E-02 50.0 1 3.3334E-02 1.4907E-02 6.16
C Variation from Pos to Pos 2.0000E-01 % Normal 99.73% 3.000 6.6667E-02 50.0 1 6.6667E-02 2.9814E-02 12.32
D Current Switching Effect 2.0000E-01 % Normal 99.73% 3.000 6.6667E-02 50.0 1 6.6667E-02 2.9814E-02 12.32
E Reference Standard 5.0000E-03 % Normal 95.45% 2.000 2.5000E-03 50.0 1 2.5000E-03 2.5000E-03 1.03
F Regulation (One hour) 2.5000E-01 % Normal 99.73% 3.000 8.3334E-02 50.0 1 8.3334E-02 8.3334E-02 34.44
G Regulation (One minute) 2.0000E-01 % Normal 99.73% 3.000 6.6667E-02 50.0 1 6.6667E-02 6.6667E-02 27.56
Variable 1
Variable 2
Correlation Coefficient from 1.0 to -1.0
Corr Values
Update Calc & Set Print Area
GUM 1995 Method Calculator
ρ of 1,2 = -0.8 3.33E-02 3.33E-02 -1.7778E-03
ρ of 1,2 = -0.8 6.67E-02 6.67E-02 -7.1112E-03
Bolded Percent of Total Values are > or = to: 25% of Expanded Uncertainty
Total Eff. Degrees of Freedom (υ)= 133.10316
Std Unc = 1.1669E-01
1.1669E-01 Std Unc
95.45%
Student's t-Distribution
To achieve the stated "Level of Confidence" using t-statistics and Effective Degrees of Freedom, the calculated k-factor = 2.019
Expanded Unc = 2.3560E-01
Level of Confidence
Normal Distribution
To achieve the stated "Level of Confidence" for an assumed "Normal" distribution, the calculated k-factor = 2.000
Expanded Unc = 2.3339E-01
Where u₁ & u₂ are correlated


Sensitivity Coefficient Wizard
1) Select Budget
20lbf_ft_Budget
Set "Major Variable ID" to V1 to Select Budget!
A) Identify formula Major Variables & Group Sub-Variables
B) Enter Measurement Model Formula
Measurement Formula Entry Wizard
Identified Major Variables
V1X: Length in feet 2 ft
V2X: Force in pounds 10 lbs
V3X:
V4X:
V5X:
V6X:
V7X:
V8X:
V9X:
V10X:
V11X:
V12X:
V13X:
V14X:
V15X:
V16X:
V17X:
V18X:
V19X:
^ Exp pi ln log sin cos tan ^4
( ) * / + - 1/x SQRT ^3
0 . 1 2 3 4 5 6 7 8 9 ^2
<- -> BackSpace Delete
Enter Measurement Formula
=V1X*V2X
20 lbf-ft
C) Update Calculation
Cancel/Exit
Enter Parameter Formula by either typing it in using your keyboard or by using the supplied buttons. Ensure you have an "=" as the FIRST character. Variables are expressed in a range from V1X to V25X.
As you encounter your listed variables, during formula entry, select them from the list on the left. The V?X name will automatically appear in the formula window. Verify formula by comparing calculated result with expected result.
If the above list contains no variables, it is because you need to select them using either the Budget Quick Connect or Select from Budget method prior to Creating Formula.

Auto-Size Worksheet

Type A Data Uncertainty Estimator

QIMT

Applied Formula

$$s_x = \sqrt{\frac{1}{\nu}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Resolution of Data

Clear ALL entries!

Type A Experimental Data			Nominal Value
			Units
Sequence of Measures	Operator(s)	Measured Values	Dev. From Nominal
Measurement 1			
Measurement 2			
Measurement 3			
Measurement 4			
Measurement 5			
Measurement 6			
Measurement 7			
Measurement 8			
Measurement 9			
Measurement 10			
Measurement 11			
Measurement 12			
Measurement 13			
Measurement 14			
Measurement 15			
Measurement 16			
Measurement 17			
Measurement 18			
Measurement 19			
Measurement 20			
Measurement 21			
Measurement 22			
Measurement 23			
Measurement 24			
Measurement 25			
Measurement 26			
Measurement 27			
Measurement 28			
Measurement 29			
Measurement 30			
Rev. Date: 24 APR 2007	Standard Deviation (uncertainty) =		0.00E+00

Freeware:

- **Gum Tree Calculator,** Measurement Standards Laboratory of New Zealand (New Zealand)
  https://gtc.readthedocs.io

- **QMSys GUM Standard (**Demo version), Qualisyst Ltd. (Bulgaria)
  http://www.qsyst.com/qualisyst_en.htm

- **GUM Workbench Pro (demo version)**, Metrodata GmbH (Germany)
  www.metrodata.de

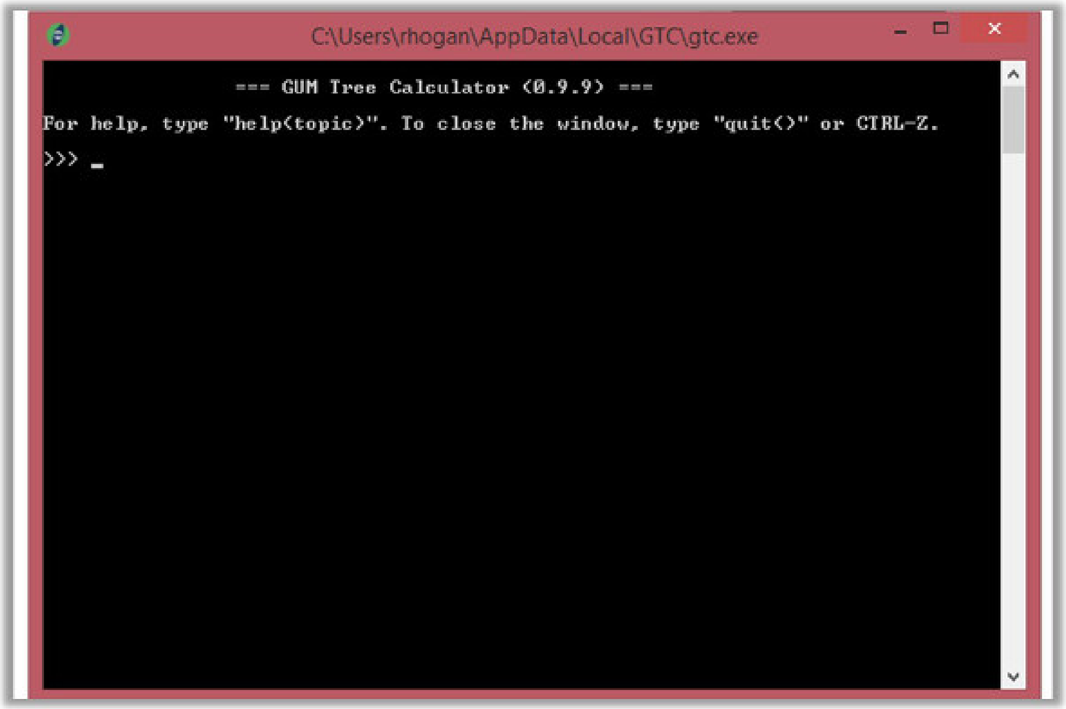
- **MUKit – Measurement Uncertainty Kit**, SYKE Finnish Environment Institute (Finland),
  https://www.syke.fi/en-US/Services/Quality_and_laboratory_services/Calibration_services_and_contract_laboratory/MUkit__Measurement_Uncertainty_Kit

Freeware:

- **NIST Uncertainty Machine**, NIST (USA)
  https://uncertainty.nist.gov/
- **Hewlett-Packard UnCal 3.2**, by Chris Grachanen (Agilent)
  https://uncertainty-calculator.software.informer.com/3.2/
- **Uncertainty Sidekick**, Integrated Sciences Group (USA)
  http://www.isgmax.com/sidekick_details.htm
- **NPL Measurement Uncertainty Software**, NPL (UK)
  https://www.npl.co.uk/resources/software/measurement-uncertainty-evaluation

GUM Tree Calculator:

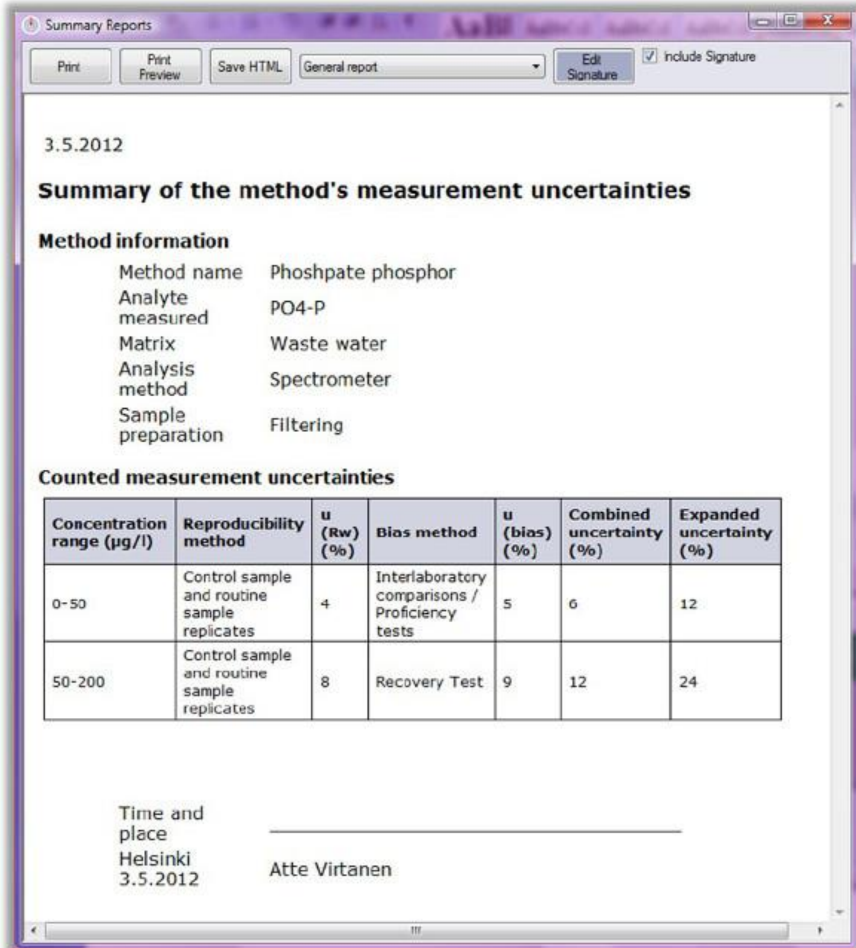
GUM Tree Calculator is a **command prompt application without a graphical user interface**. So, knowledge of programming (in Python) is a must for you to make full use of this uncertainty calculator.



MuKIT Measurement Uncertainty KIT:

This software was developed to estimate uncertainty **for chemical, biological, and life science laboratories.**

(Seems to be hard to install!)



3.5.2012

Summary of the method's measurement uncertainties

Method information

Method name	Phoshpate phosphor
Analyte measured	PO4-P
Matrix	Waste water
Analysis method	Spectrometer
Sample preparation	Filtering

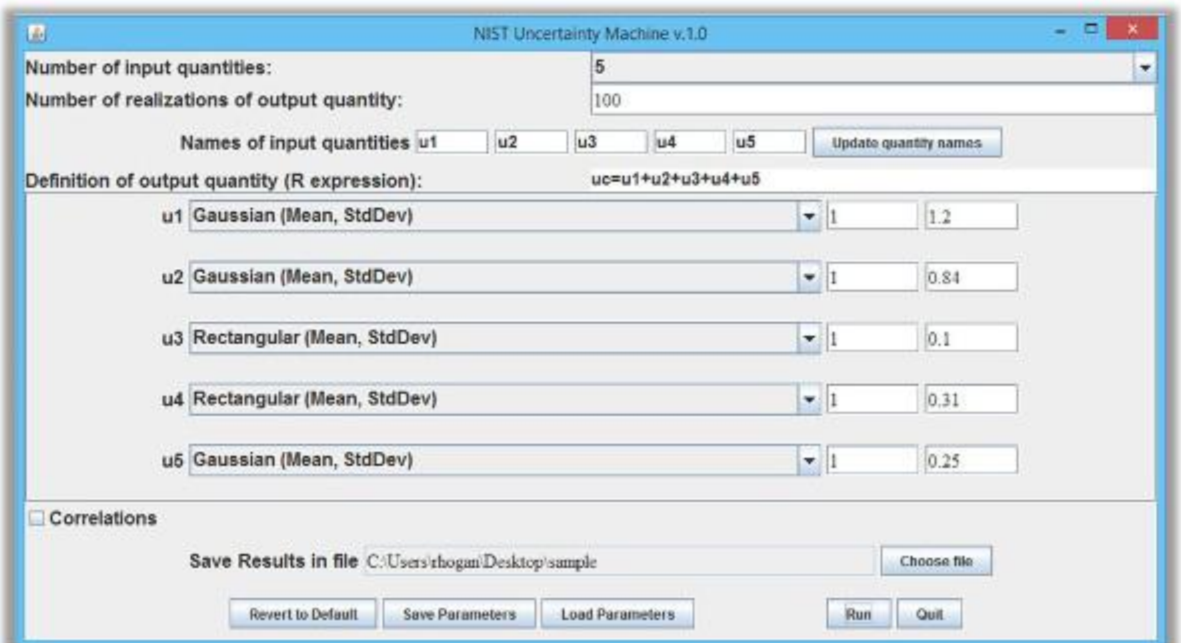
Counted measurement uncertainties

Concentration range (µg/l)	Reproducibility method	u (Rw) (%)	Bias method	u (bias) (%)	Combined uncertainty (%)	Expanded uncertainty (%)
0-50	Control sample and routine sample replicates	4	Interlaboratory comparisons / Proficiency tests	5	6	12
50-200	Control sample and routine sample replicates	8	Recovery Test	9	12	24

Time and place	______________________
Helsinki 3.5.2012	Atte Virtanen

NIST Uncertainty Machine:

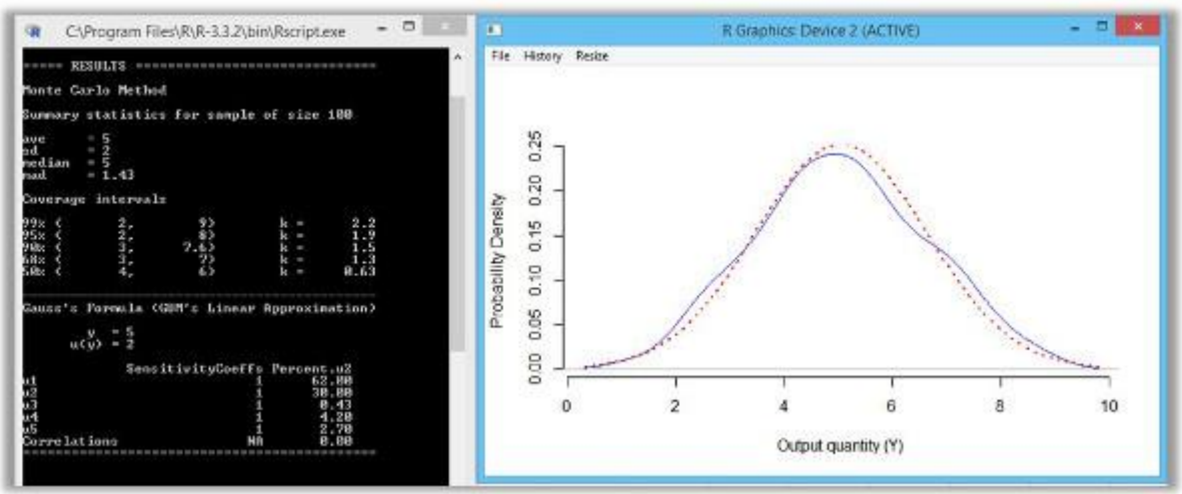
To use the NIST Uncertainty Machine software, you will need to **download R software first** ("R" represents a open source statistics software).
For calculations according to GUM. Also Monte Carlo calculations feasible.



NIST Uncertainty Machine:

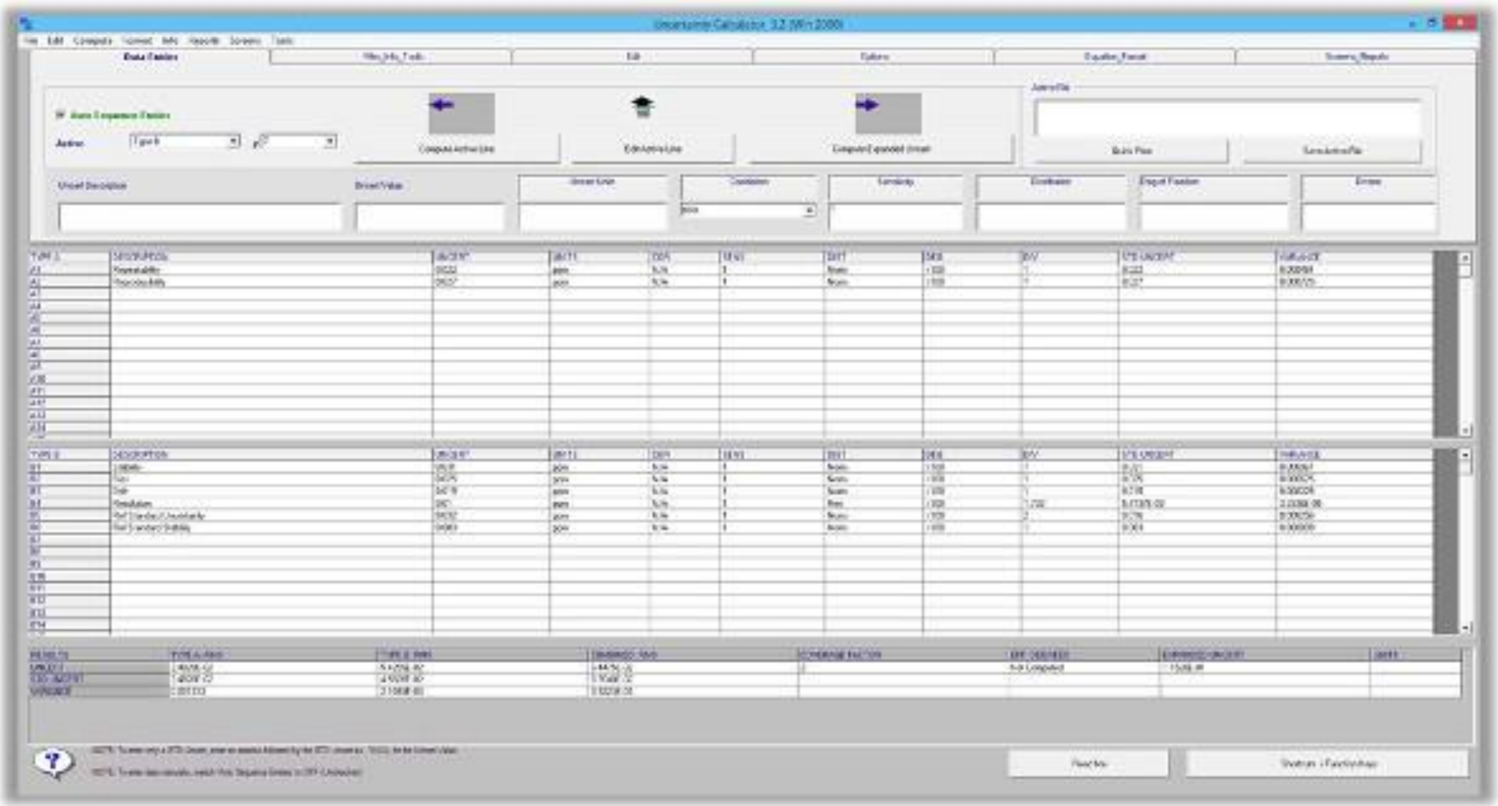
Key Takeaways

- Plain user interface
- Not many options
- Easy to use after successful installation of “R“
- Better suited for uncertainty analysis of equations
- Confusing results screen with a nice graph
- Results truncated to whole numbers



Hewlett Packard UnCal 3.2:

UnCal 3.2 is a simple uncertainty calculator that is not easy to use and not so easy to learn.



Hewlett Packard UnCal 3.2:

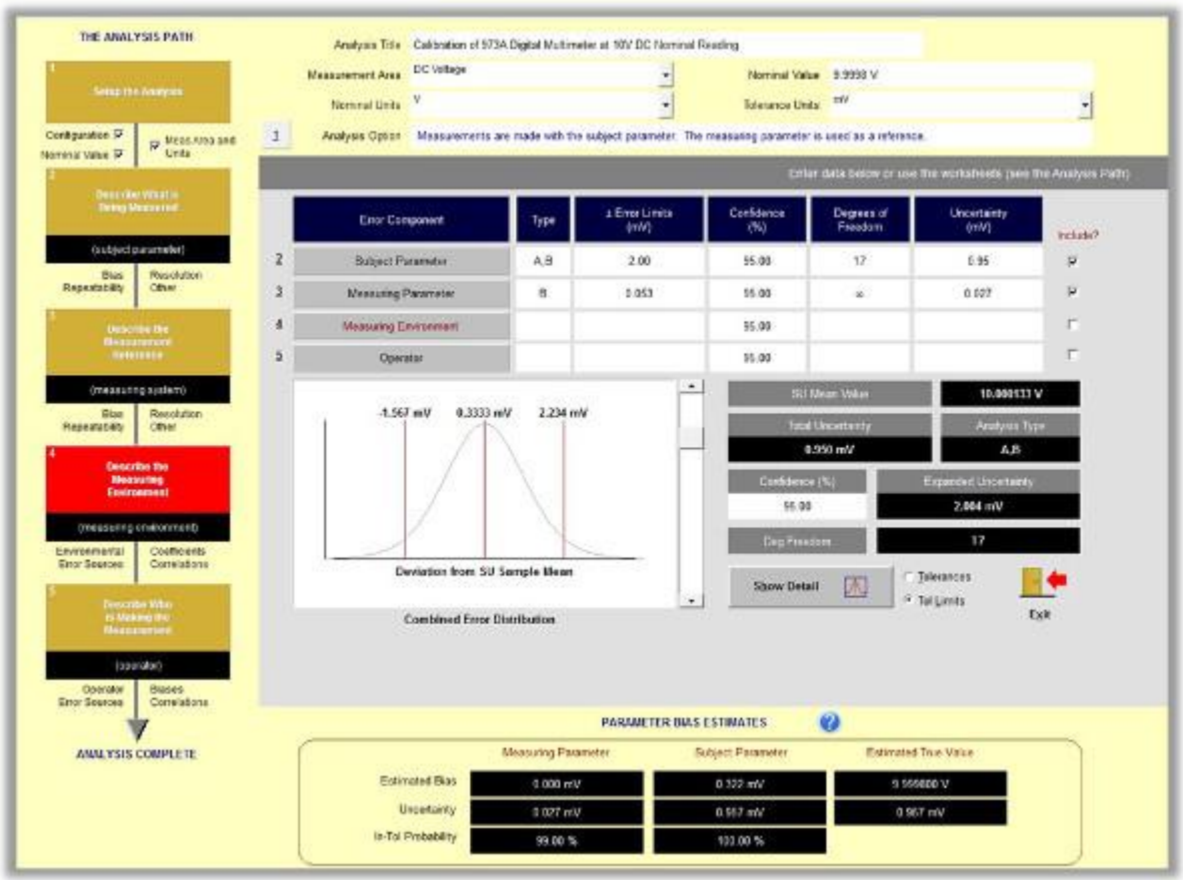
Key Takeaways

- Dated Technology (Windows 2000 application)
- Not so easy to use
- Not so easy to learn
- Plenty of Functions

If you are looking **to calculate uncertainty as a beginner** (and don't want to be overwhelmed with too many features) or just want a simple uncertainty calculator that works, **UnCal 3.2 could be an option**.

Uncertainty Sidekick:

According to the developer, **the software is designed to guide you through the process** of estimating uncertainty in measurement results.



Uncertainty Sidekick:

Key Takeaways

- Great user interface
- Great user experience
- Instructions to guide you through the process
- Easy to Use

It has a **lot of features** (more than you will probably need). However, what I liked most about it was the **presentation of results** and the **diagram to help guide users through the process**.

This is a **great feature for beginners or any user** for that matter.

Summary of Main Software Features and Capabilities

Features and Capabilities	**Uncertainty Sidekick 1.0**	**Uncertainty Sidekick Pro 1.0**	**Uncertainty Analyzer 3.0**
Single-User Price (USD)	Free via download	$295 (Software CD & User Manual)	$995 (Software CD & User Manual)
Analysis of Direct Measurements	ü	ü	ü+
Analysis of Multivariate Measurements		ü	ü+
Analysis of Measurement Systems[1]			ü
Uncertainty Budget Table	ü	ü	ü+
Maximum Number of Error Sources Allowed	15	40	Over 1,000

NPL Measurement Uncertainty Software:

Using the software **requires you to install MATLABS's Component Runtime** (MCR) libraries, which are free of charge.

Enables the user to carry out calculations according to the GUM and also doing Monte Carlo Calculations.

My personal conclusions:

For beginners and professionals:

GUMsim (quo data, 840 €)

GUM Professional (QMsys, 1000 €)

GUM workbench (Metrodata, 2023 €)

All the other options for professionals resp. specialists!

Freeware: Eventually Hewlett Packard Uncal 3.2